Heat Transfer :-

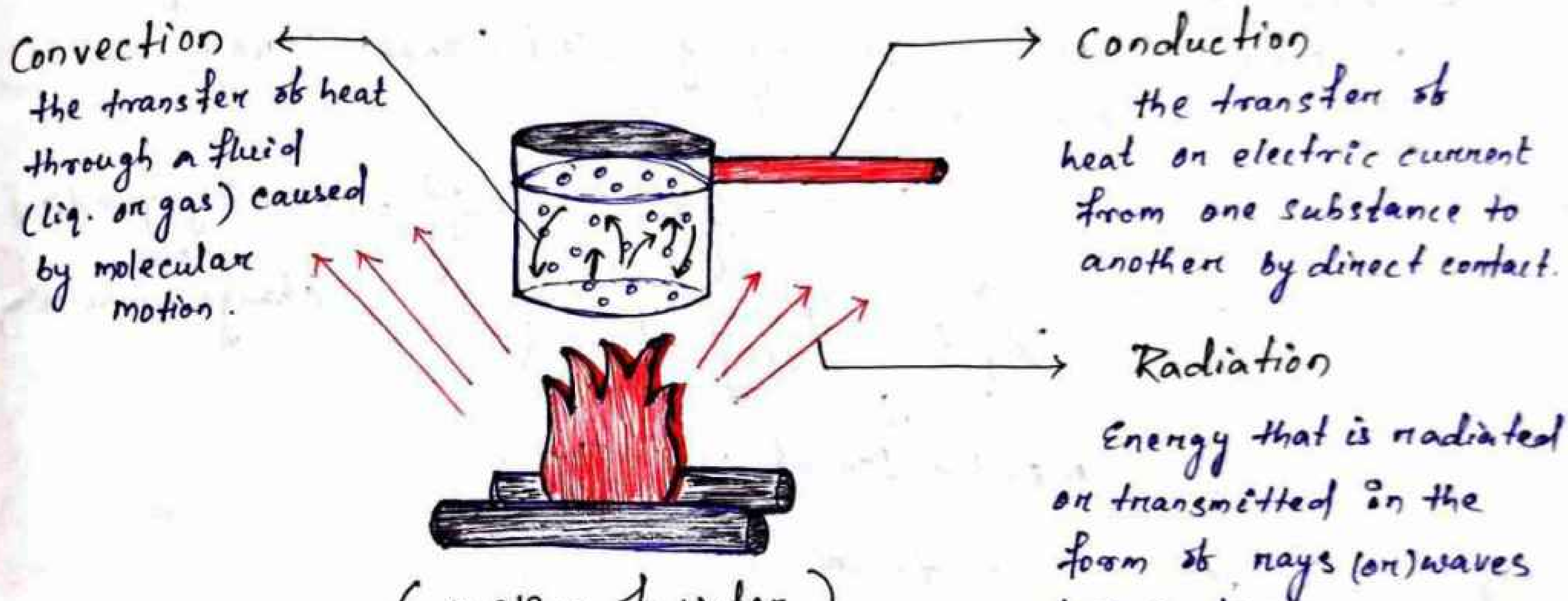
It is defined as the process in which the particles are moved from the region of higher temp. to lower temp.

3 modes of heat transfer — (i) conduction
(ii) covection
(iii) Radiation

(i) **Cond^n** :- Heat transfer from one particle to another particle by direct contact.

(ii) **Convection** :- It involves the ~~mom~~ movement of large fluid.

(iii) **Radiat^n** :- Heat transfer of energy through electromagnetic waves.



(Boiling of water)

What is Heat Transfer:-

Ans:- It is defined as the process in which the particle are move form the region of high temperature to low temperature called Heat Transferr.

Modes of Heat Transfer:-

→ There are Three Type of Heat Transfer

1. Conduction Ex:- solid
2. Convection Ex:- liquid
3. Radiation Ex:- Vacuum, gas

① **Conduction:-** Conduction is the process of Heat transfer in which heat transfer form one molecule of the body to another molecule without actual motion of the body.

→ Heat transfer by solid medium.

Ex:- (Iron Rod, is Heated by one end)

② **Convection:-** Convection is the process of Heat transfer in which heat transfer of one particle to fluid to another fluid particle by the actual motion of heated particle.

→ Heat transfer by liquid medium.

Ex:- Boiling of water.

③ **Radiation:-** Radiation is the process of Heat transfer form a hot body to a cooled body in straight line without affecting in the interface.

→ Heat transfer by vacuum and gaseous medium.

Ex:- sun light to earth
Heat from the sun reaches to the earth.

Fluid :- The substances which can flow easily are called fluid, of (higher level to lower level).

Ex:- water, oil etc

Types of fluid flow :-

① steady and unsteady flow
② uniform and Non-uniform flow
③ laminar and Turbulent flow
④ compressible and In-compressible flow
⑤ Rotational and irrotational flow
⑥ one, Two and Three dimensional flow

① **steady flow** :-

It is defined as that type of flow in which the flow characteristics like velocity, pressure, density at a point does not change with respect to time.

mathematically :-

$$\frac{dP}{dt} = 0, \quad \frac{dV}{dt} = 0, \quad \frac{d\rho}{dt} = 0$$

* **unsteady flow:—** It is the Type of flow in which the velocity, pressure and density at a point changes with respect to time is called unsteady flow.

mathematically:—

$$\frac{dp}{dt} \neq 0, \quad \frac{dv}{dt} \neq 0, \quad \frac{d\rho}{dt} \neq 0$$

(2) **uniform flow:—**

It is defined that Type of flow in which the velocity at any given time does not change with respect to space. (length of flow in the direction).

mathematically:—

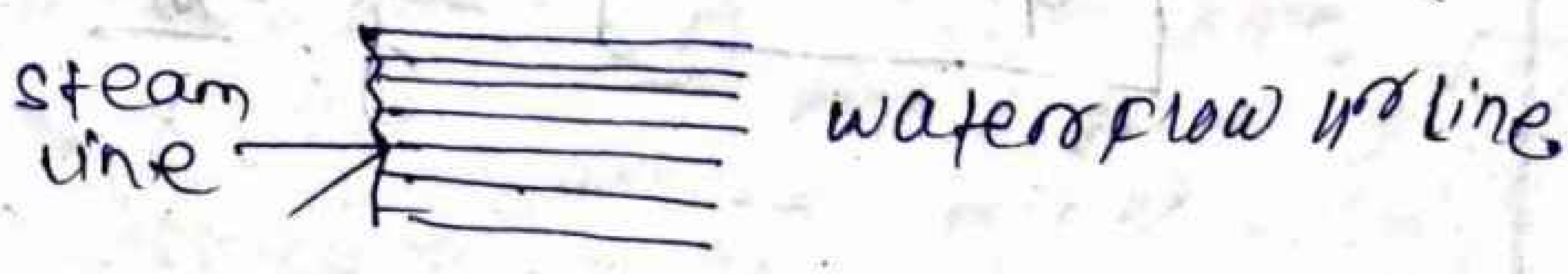
$$\frac{dv}{ds} = 0$$

* **Non-uniform:—** It is the Type of flow in which the velocity at given ~~cha~~ time changes with respect to the space

$$\frac{dv}{ds} \neq 0$$

(3) **laminar flow:—** It is that Type of flow in which the fluid particle moves along well defined path or stream line are straight and parallel.

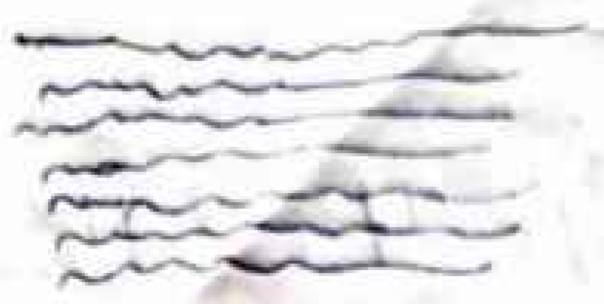
→ This Type of flow is also called stream lines flow or viscous flow.



* Turbulant flow :-

→ It is that types of flow in which the fluid particle move in zigzag flow called turbulant.

→ velocity is increase diameter of pipe is increase the viscocity of fluid is increase



(4) compressible flow :- 9/08/2023

→ In which flow the density of fluid changes from one point to another point w.r.t time is called compressible flow.

Mathematically :-

$$\frac{d\rho}{dt} \neq 0$$

$\boxed{\rho\text{-density}}$

ρ = mass/volume

$\rho = \frac{m}{v}$ [change in volume but constant mass.]

* In compressible flow :-

In which flow the density of fluid does not changes from one point to another point w.r.t time is called in compressible flow.

$$\boxed{\frac{d\rho}{dt} = 0}$$

[No mass change and volume]

$\rho = \frac{m}{v}$

(5) Rotational flow :-
In which fluid flow molecules are rotates its axis and moving along its stream line is called Rotational flow.

stream line :- straight line & parallel

* Irrotational flow :-
In which fluid flow molecule are not rotate an its axis and moving along its stream line is called irrotational flow.

(6) one Two & Three dimensional flow :-
The Term one two and three dimensional flow refers to the no. of space co-ordinated required to describe a flow.

* one dimensional :- 1-D.F
In this flow the velocity is a function of time and one rectangular space co-ordinate only.

* 2-D flow :-
In this flow the velocity is a function of time and two rectangular space co-ordinate only.

* 3-D flow :- In this flow the velocity is a function of time and Three rectangular, space co-ordinate only.

TYPES OF FLUID :—

* Types of fluid :—

The fluids are classified into 5 type

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non- Newtonian fluid
5. Ideal-plastic fluid

(1) Ideal fluid :— A fluid which is in-compressible, and this having no viscosity is know as an Ideal fluid.

→ Ideal fluid is only an imaginary fluid.

$$\boxed{\text{Shear stress } \tau = 0} \quad \text{TOU}$$

(2) Real fluid :— A fluid which possesses viscosity is know as Real fluid.

→ All fluids in actual practice are real fluids.

$$\boxed{\tau \neq 0}$$

(3) Newtonian fluid :—

(change in velocity)

$$\tau \propto \frac{du}{dy} \Big\} \text{velocity gradient}$$

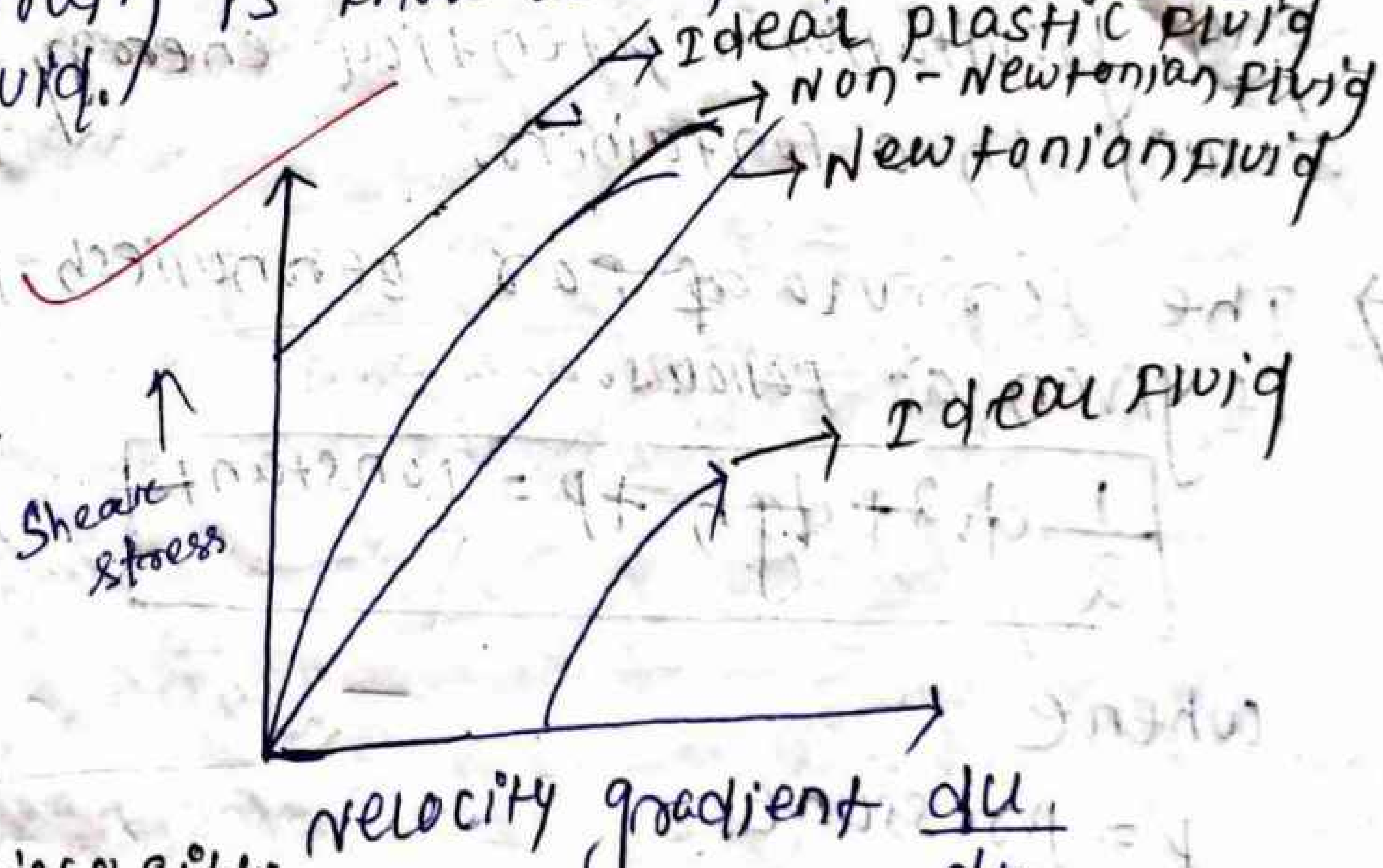
$$\boxed{\tau = \mu \cdot \frac{du}{dy}} \quad \mu = \text{constant.}$$

→ Newton law of viscosity

A real fluid in which the shear stress is directly proportional to the velocity gradient [shear strain] is know as Newtonian fluid.

(4) Non-Newtonian fluid:–
A real fluid in which the shear stress is not directly proportional to the velocity gradient is know as non-Newtonian fluid.

(5) Ideal-plastic fluid:–
A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain is know as an ideal plastic fluid.



Viscosity:–
viscosity is the process of Resistance to flow.

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Bernoulli's Law :—

Bernoulli's principle state that the total mechanical energy of the moving fluid comprising the gravitational potential energy of elevation, the energy associated with the fluid pressure and the kinetic energy of the fluid motion remains constant.

→ It can be derive from the principle of conservation of energy.

→ Bernoulli's equation formula is the relationship between pressure kinetic energy and gravitational potential energy of a fluid in a container.

→ The formula ~~of~~ for Bernoulli's principle is given as follows.

$$\boxed{\frac{1}{2} d v^2 + d g h + p = \text{constant}}$$

where

p = pressure

v = velocity

d = density

h = heigh

g = Acceleration due to gravity $\boxed{9.81 \text{ m/s}^2}$

→ This equation give the balance between pressure, velocity and height.

→ **Bernoulli's equation derivation :—**

→ consider a non-uniform pipe with varying diameter and height through which an incompressible fluid is flowing.

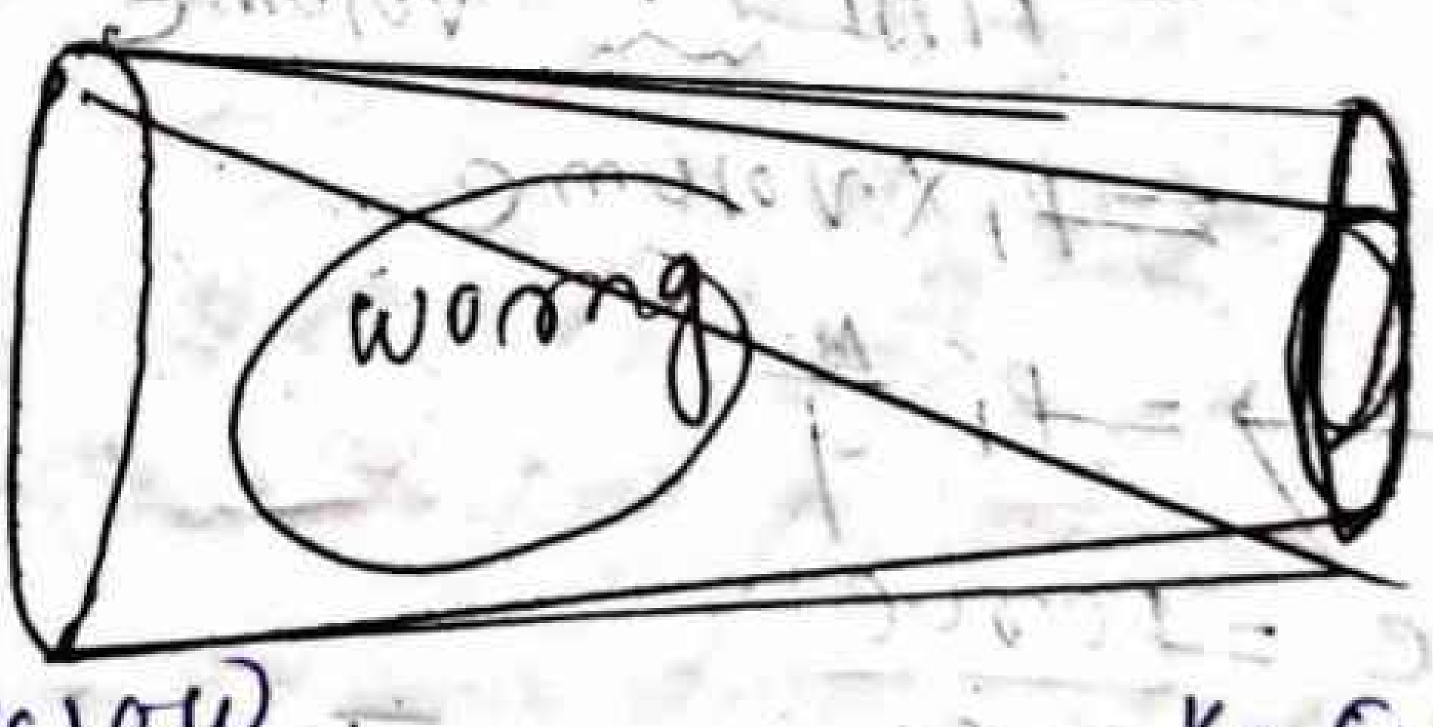
→ The relationship between the area of crosssection (A).

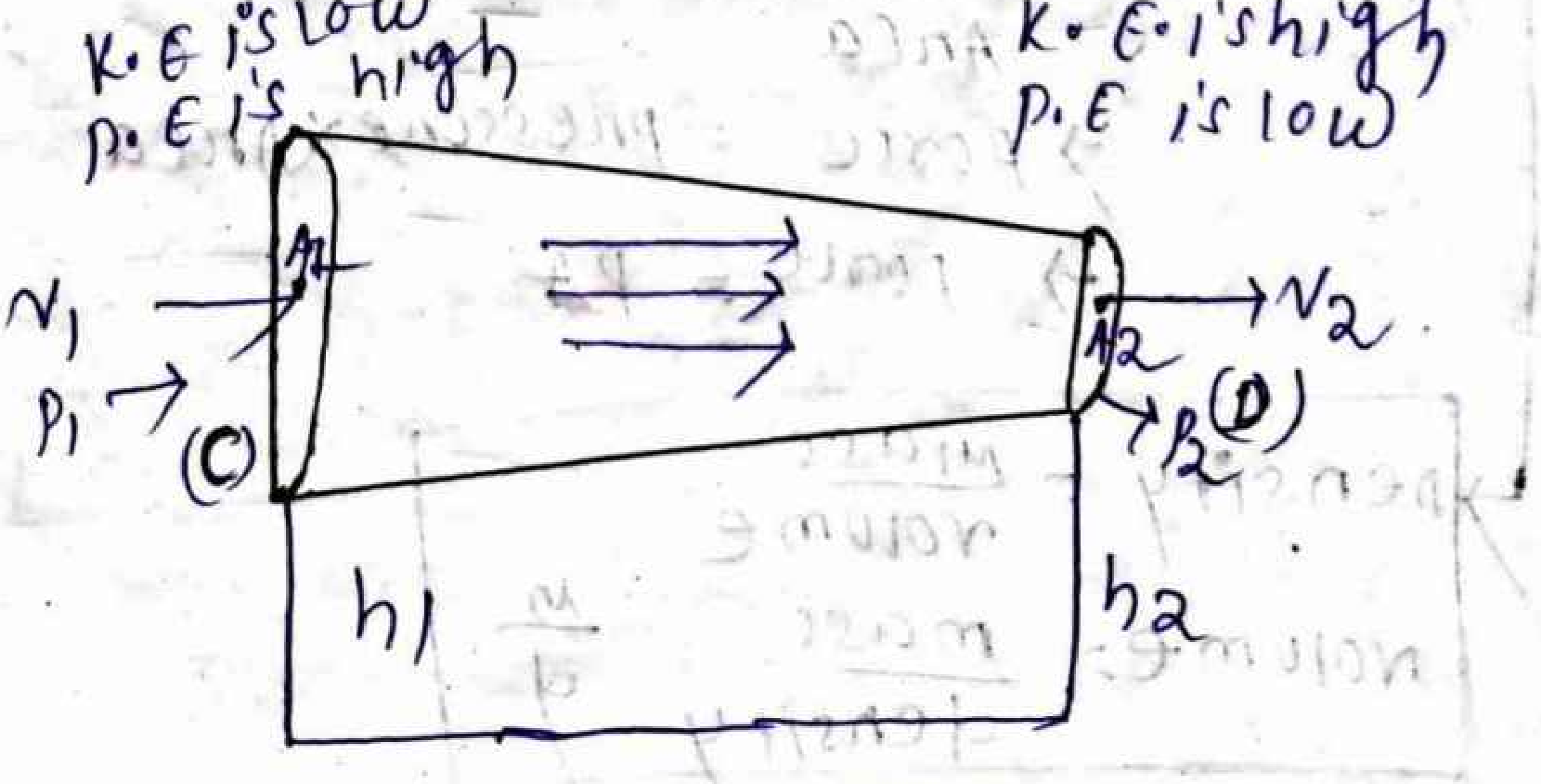
→ The flow speed (v) :

→ Height from the ground (h) and

→ the pressure (P).

→ At two different points C & D are given in the figure below





$v_2 > v_1 \rightarrow A_2$ is low and A_1 is high

$\rightarrow$ K.E $= \frac{1}{2} mv^2$

$\rightarrow$ P.E $=$ m.g.h

$\rightarrow$ work done $=$ P.d.V

$\rightarrow$ Increase in K.E from C to D point $=$

$$= \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 \quad \text{——— (I)}$$

$\rightarrow$ Decrease in P.E from C to D point $=$

$$mgh_1 - mgh_2 \quad \text{——— (II)}$$

work done in point 'C' $=$ force $\times$ displacement

$$= P_1 A_1 \times v_1 t$$

$$= P_1 \underbrace{A_1 v_1 t}_{\text{volume}}$$

$$= P_1 \times \text{volume}$$

$$= P_1 \frac{m}{d}$$

pressure $= \frac{\text{force}}{\text{Area}}$

$\Rightarrow$ force $=$ pressure $\times$ Area

$\Rightarrow$ force $=$ PA

Density $= \frac{\text{mass}}{\text{volume}}$

volume $= \frac{\text{mass}}{\text{density}} = \frac{m}{d}$

similarly at point $P_2\frac{m}{d}$

Net work done $= P_1\frac{m}{d} - P_2\frac{m}{d}$ —— (3)

Applying energy conservation

Increase in K.E = Decrease in P.E + work done

$$\Rightarrow \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = mgh_1 - mgh_2 + P_1\frac{m}{d} - P_2\frac{m}{d}$$

$$\Rightarrow \underbrace{\frac{1}{2}mv_1^2 + mgh_1 + P_1\frac{m}{d}}_{\text{point 'C'}} = \underbrace{\frac{1}{2}mv_2^2 + mgh_2 + P_2\frac{m}{d}}_{\text{point 'D'}}$$

[The equation is divide by $\frac{m}{d}$ both side]

$$\Rightarrow \frac{\frac{1}{2}mv_1^2 + mgh_1 + P_1\frac{m}{d}}{\frac{m}{d}} = \frac{\frac{1}{2}mv_2^2 + mgh_2 + P_2\frac{m}{d}}{\frac{m}{d}}$$

$$\Rightarrow \frac{1}{2}mv_1^2 + mgh_1 + P_1\frac{m}{d}\left(\frac{d}{m}\right) = \frac{1}{2}mv_2^2 + mgh_2 + P_2\frac{m}{d}\left(\frac{d}{m}\right)$$

$$\Rightarrow \left(\frac{1}{2}\cancel{m}v_1^2 \times \frac{d}{\cancel{m}}\right) + \left(\cancel{m}gh_1 \times \frac{d}{\cancel{m}}\right) + \left(P_1\frac{\cancel{m}}{\cancel{d}} \times \frac{\cancel{d}}{\cancel{m}}\right) = \left(\frac{1}{2}\cancel{m}v_2^2 \times \frac{d}{\cancel{m}}\right) + \left(\cancel{m}gh_2 \times \frac{d}{\cancel{m}}\right) + \left(P_2\frac{\cancel{m}}{\cancel{d}} \times \frac{\cancel{d}}{\cancel{m}}\right)$$

$$\Rightarrow \frac{1}{2}v_1^2 d + gh_1 d + P_1 = \frac{1}{2}v_2^2 d + gh_2 d + P_2$$

$$\Rightarrow \boxed{\frac{1}{2}dv^2 + dgh + P = \text{constant}}$$

$\boxed{\text{L.H.S} = \text{R.H.S}}$ proved

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EQUATIONS OF MOTION :-

According to newton 2nd law of motion The net force F_x, acting on a fluid element in the direction of 'x' is equal to mass 'm' of the fluid element multiplied by acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m \cdot a_x$$

In the fluid flow the following force are present :

1. F_g gravity force
2. F_p The pressure force
3. F_v force due to viscosity
4. F_t force due to turbulance
5. F_c force due to compressiblity.

Thus in eqⁿ, the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

If the flow is assumed to be ideal viscous force (F_v) is zero and equation of motion are as Euler's equation of motion.

1st part — EULERS EQUATION OF MOTION :—

This is equation of motion in which the forces due to gravity and pressure are taken into consideration.

→ This derived by considering the motion of a fluid element along a streamline.

→ consider a stream line in which flow is taking place in 's' direction.

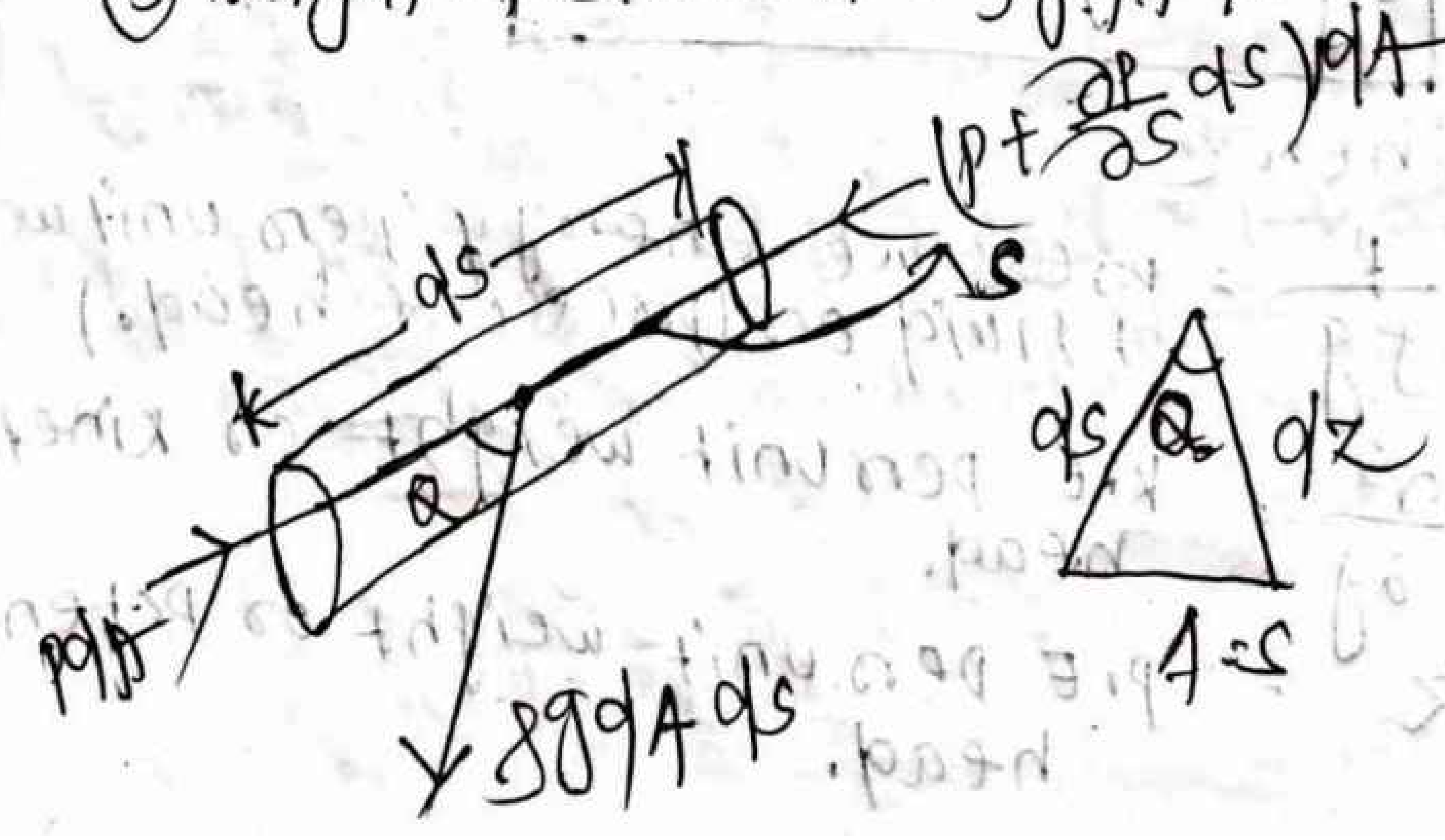
→ consider a cylindrical element of crossection dA and length ds.

→ The forces acting on the cylindrical element are

1. pressure force pdA in the direction of flow.
2. pressure force,

$$\left(p + \frac{\partial p}{\partial s} ds\right) dA$$ opposite to the direction of flow.

3. weight of element $\rho g dA ds$



let θ is the angle between the direction flow and the line of action of the weight of element

→ The resultant force on the fluid element in the direction of 's' must be equal to the mass of fluid element ~~into~~ × acceleration in the direction 's'

$$\Rightarrow p\,dA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g\, dA\, ds \cos\theta$$

(net force) $= \rho\, dA\, ds \times a_s$

* $p\,dA$ → upper direction
$\left(p + \frac{\partial p}{\partial s} ds\right) dA$ → lower direction
~~...~~ $\rho g\, dA\, ds \cos\theta$ = direction of flow

$\rho\, dA\, ds \times a_s$ = mass

* where
a_s is the acceleration in the direction of 's' $a_s = \frac{dv}{dt}$

* where
v is the function of s and t

$$= \frac{\partial v}{\partial s} \times \frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$= \frac{v\,\partial v}{\partial s} + \frac{\partial v}{\partial t} \left\{ \frac{ds}{dt} = v \right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\Rightarrow a_s = \frac{v\,\partial v}{\partial s}$$

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⇒ $\cancel{pdA} - \cancel{pdA} - \frac{\partial p}{\partial s} ds \cdot dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \frac{\partial v}{\partial s}$

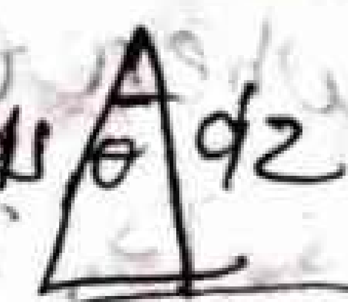
⇒ $-\frac{\partial p}{\partial s} ds \cdot dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \times v \frac{\partial v}{\partial s}$

Diveded entire eauation by $\rho\, dA\, ds$

⇒ $\frac{-\frac{\partial p}{\partial s} \cancel{ds} \cdot \cancel{dA}}{\rho \cancel{dA}\, \cancel{ds}} - \frac{\cancel{\rho} g \cancel{dA}\, \cancel{ds} \cos\theta}{\cancel{\rho} \cancel{dA}\, \cancel{ds}} = \frac{\cancel{\rho} \cancel{dA}\, \cancel{ds} \times v \frac{\partial v}{\partial s}}{\cancel{\rho} \cancel{dA}\, \cancel{ds}}$

⇒ $-\frac{\partial p}{\partial s} \times \frac{1}{\rho} - g \cos\theta = v \frac{\partial v}{\partial s}$

$= -\frac{1}{\rho} \frac{\partial p}{\partial s} - g \cos\theta = \frac{v\, \partial v}{\partial s}$



$= -\left(\frac{1}{\rho} \frac{\partial p}{\partial s} + g \cos\theta\right) + v \frac{\partial v}{\partial s} = 0$

$\cos\theta = \frac{dz}{ds}$

⇒ $\frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$

partially = ∂ or d
so ∂ is d

⇒ $\frac{1}{\partial s}\left(\frac{dp}{\rho} + g\, dz + v\, dv\right) = 0$

⇒ $\frac{1}{ds}\left(\frac{dp}{\rho} + g\, dz + v\, dv\right) = 0$

⇒ $\boxed{\frac{dp}{\rho} + g\, dz + v\, dv = 0}$

Bernoulli's equation from Euler's eqn of motion :-

Euler's equation's of motion :-

2nd part $\boxed{\frac{dp}{\rho} + g\,dz + v\,dv = \text{const}}$

→ Bernoulli's eqn is obtained by Integrating the Euler's eqn of motion as

$$\int \frac{dp}{\rho} + \int g\,dz + \int v\,dv = \text{Const.}$$

$$= \frac{1}{\rho}\int dp + d\int gz + d\int dv.dv$$

$$\Rightarrow \frac{1}{\rho} \times p + gz + \frac{v^2}{2} = \text{const.}$$

[where ρ = const. Incompressible]

Dividing 'g' in above eqn :-

$$\Rightarrow \frac{p}{\rho g} + \frac{gz}{g} + \frac{v^2}{2g} = \text{const.}$$

$$\Rightarrow \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{const.}$$

$$\Rightarrow \boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{const.}}$$

where

* $\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or (pressure head.)
* $\frac{v^2}{2g}$ = K.E per unit weight or kinetic head.
* z = P.E per unit weight or potential head.

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3rd

PRACTICAL APPLICATION OF BERNOULLI'S EQUATION:

Bernoulli's equation is applied in all problem of incompressible fluid flow where energy consideration are involved. But we shall consider its application to the following measuring devices :-

1. venturimeter
2. orifice meter
3. pitot - tube

(1) venturimeter :— A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consist three parts :

1. A short converging part
2. Throat and
3. Diverging part, it is based on the principle of Bernoulli's equation.

Expression for rate of flow through venturimeter :—

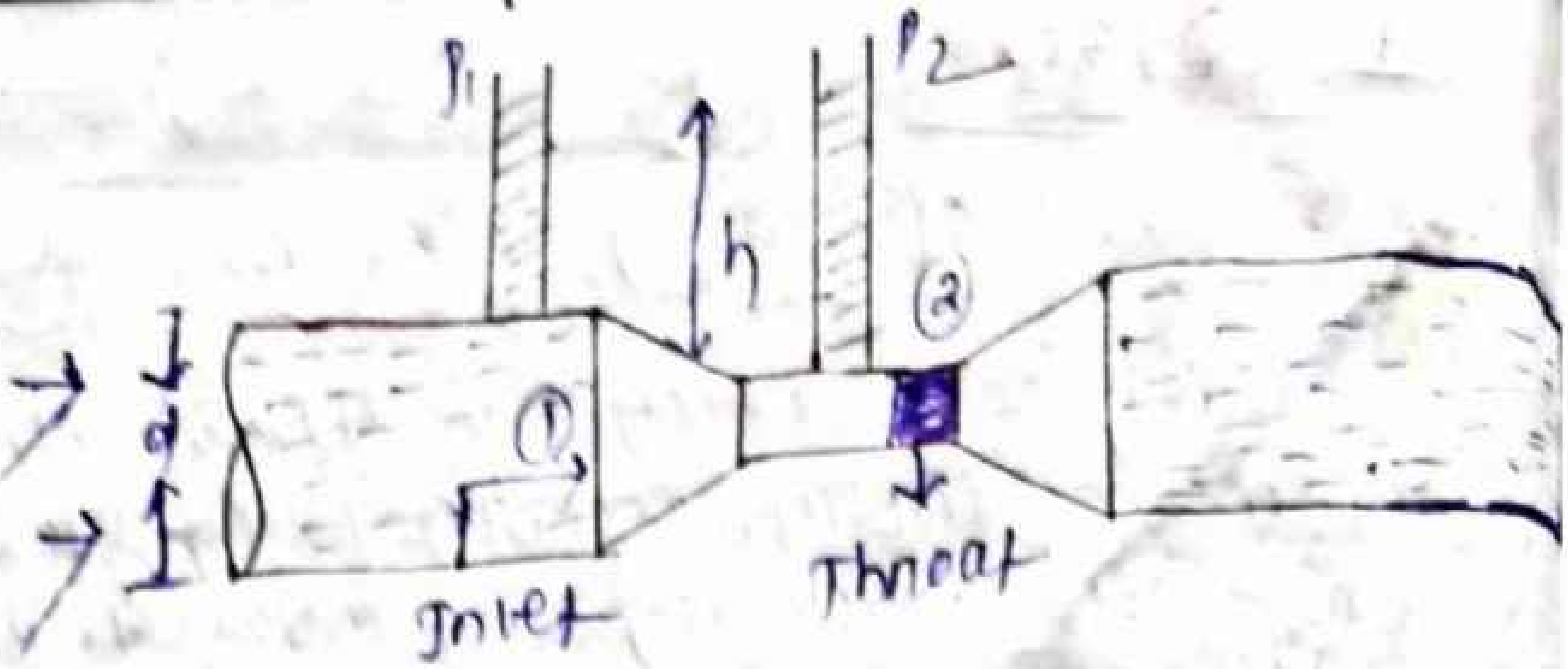
considers a venturimeter fited in horizontal pipe which fluid is flowing (say water) as shown in fig.

let d_1 = diameter at inlet or at section

P_1 = pressure at section

v_1 = velocity of fluid at section

a = area at section $\frac{\pi}{4} d_1^2$



and $d_2 . P_2 . v_2 . a_2$ are corresponding values at section (2). Applying Bernoulli equation at section (1) and (2) we get

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \quad \text{---(1)}$$

As pipe is horizontal hence $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\text{or} \quad h \quad \frac{P_1 - P_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of pressure head at section 1 and 2 it is equal to h or $\frac{P_1 - P_2}{\rho g} = h$

Substituting this value of $\frac{P_1 - P_2}{\rho g}$ in the above equation we get.

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{---} (2)$$

Now applying continuity equation at section 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in eq^n — (2)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] =$$

$$\frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or $$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}}$$

$$= \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \cdots (3)$$

eq^n(6,7) give the discharge under ideal condition and is called theoritical discharge. Actual discharge will be less than theoritical discharge.

$$\therefore Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad (iv)$$

where

C_d = co-efficient of ~~venturiment~~ venturimeters and its value is less than.

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* Orificemeter or orifice plate :-

→ It's a device used for measuring the rate of flow of a fluid through a pipe. It's a cheaper device as compared to venturimeter.

→ It also work on the same principle as that venturimeter.

→ which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe. Through it may vary form 0.4 to 0.8 time the pipe diameter.

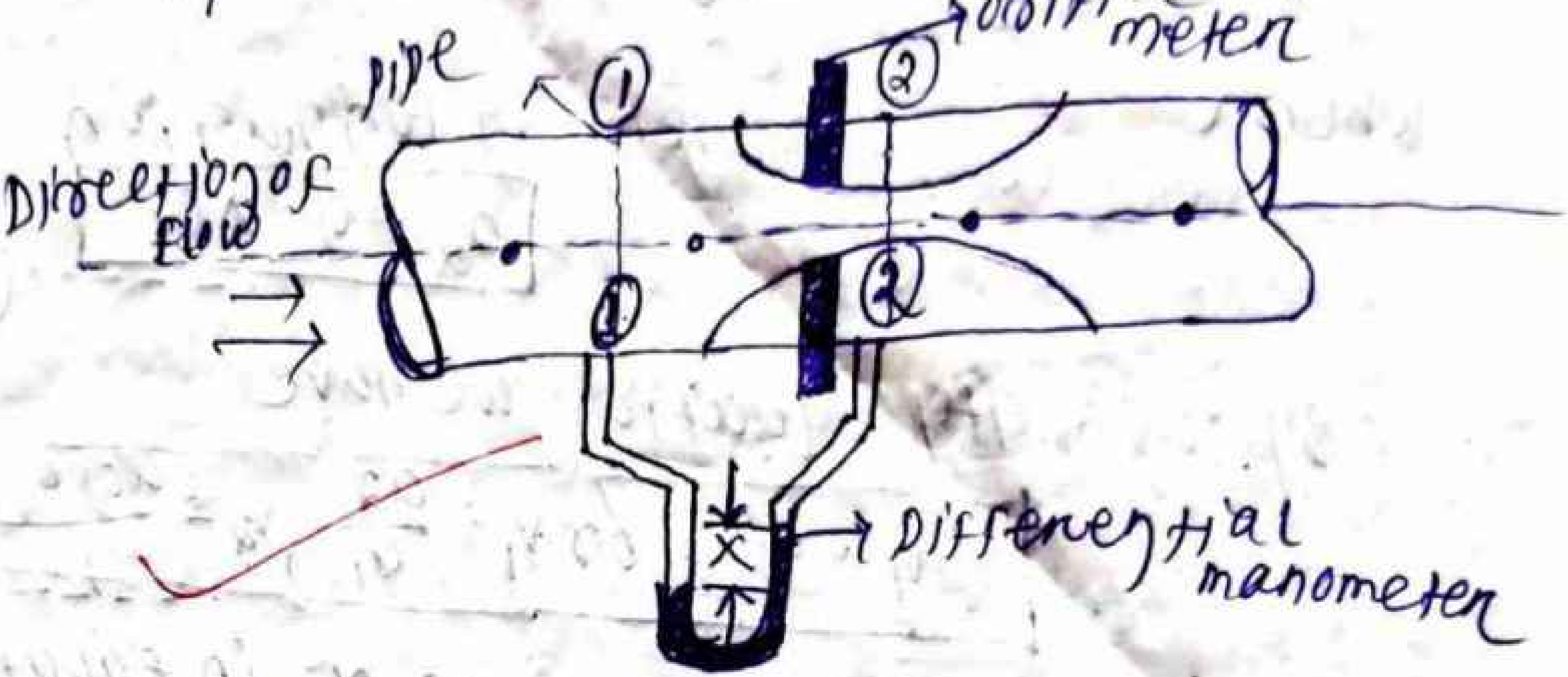
→ It consist of circular flat plate which has a circular sharp edged hole called orifice which is concentric with pipe

→ A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 time the pipe diameter upstream from the orifice plate, and at section (2), which is it at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at sec.(1)

v_1 = velocity at sec(1)

a_1 = area of pipe at sec(1) and



p_2, v_2, a_2 are corresponding value at sec(2)

Applying Bernoulli's equation at section (1) and (2) we get.

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

$$\text{or } \left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

$$\text{but } \left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=h$$

where h = Differential head

$$h=\frac{v_2^2}{2g}-\frac{v_1^2}{2g} \text{ or } 2gh=v_2^2-v_1^2$$

$$v_2=\sqrt{2gh+v_1^2} \quad \text{--- (1)}$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$$\boxed{a_2 = a_0 \times C_c} \quad \text{--- (ii)}$$

By continuity equation we have

$$\boxed{a_1 v_1 = a_2 v_2 \text{ or } v_1 = \frac{a_2}{a_1}, v_2 = \frac{a_0 C_c}{a_1} v_2} \quad \text{--- (iii)}$$

substituting the value of v_1 in equation (i) we get $v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$

or $v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2$

or $v_2^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$

$$\therefore v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad \left[\because a_2 = a_0 C_c \text{ from (ii)}\right]$$

$\therefore$ The discharge

$$Q = v_2 \times a_2 = v_2 \times a_0 C_c$$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad \text{--- (iv)}$$

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The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

substituting this value of C_c in eq^n (iv) we get

$$Q = a_0 + C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d\, a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d\, a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

where C_d = Co-efficient of dischange for orifice meter.

The co-efficient of dischange for orifice meter is much smaller than that for a ventorimeter.

PITOT-TUBE :— 25/08/2023

→ It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

→ It is based on the principle that if the velocity of flow at a point become zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.

→ In its simplest form, the pitot-tube consist of a glass tube, bent at a right angle as shown figure.

→ The lower end which is bent through 90° is directed in the up-stream direction as shown as fig.

→ The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy.

→ The velocity is determined by measuring the rise of liquid in the tube.

→ Consider two point ① & ② at the same level in a such a way that point ② is just as the inlet of the pitot tube and point ① is far away from the tube.

p_1 = Intensity of pressure at point ①
v_1 = velocity of flow at (1)
p_2 = pressure at point (2)
v_2 = velocity at point (2) which is zero

H = depth of tube in the liquid

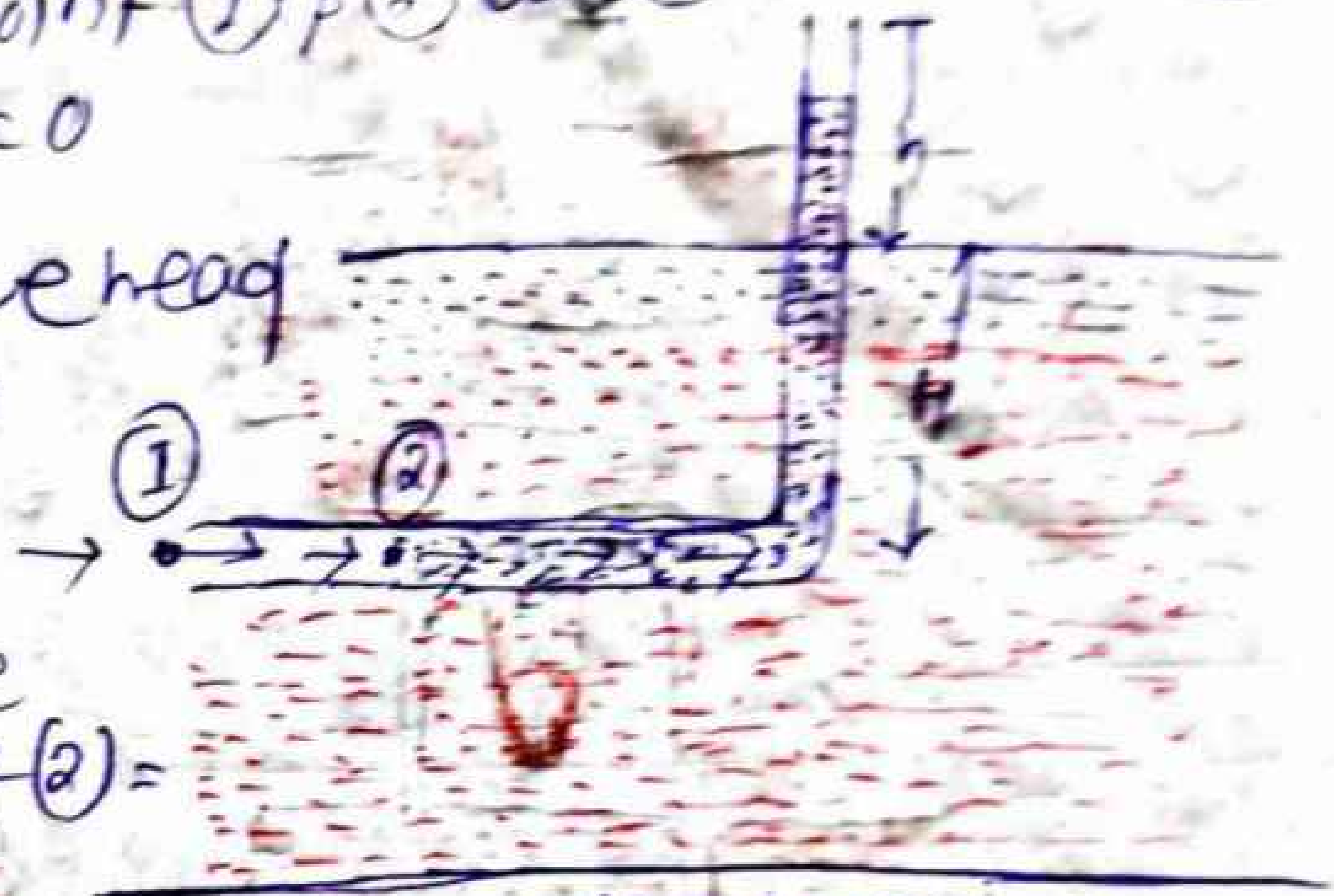
h = rise of liquid in the tube above the free surface

Applying Bernoulli's equation point (1) and point (2) we get

$$\boxed{\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2}$$

But $z_1 = z_2$ as point (1) & (2) are the same line and $v_2 = 0$

$\frac{p_1}{\rho g}$ = pressure head at (1) = H



pitot-tube

$\frac{p_2}{\rho g}$ = pressure head at (2) = $(h+H)$

substituating these value, we get

$$\therefore H + \frac{v_1^2}{2g} = (h+H)$$

$$\therefore h = \frac{v_1^2}{2g} \text{ or } \boxed{v_1 = \sqrt{2gh}}$$

This is theoritical velocity. Actual velocity is given by

$$\boxed{(v_1)_{act} = C_v\sqrt{2gh}}$$

where C_v = co-efficient of pitot-tube

$\therefore$ velocity at any point

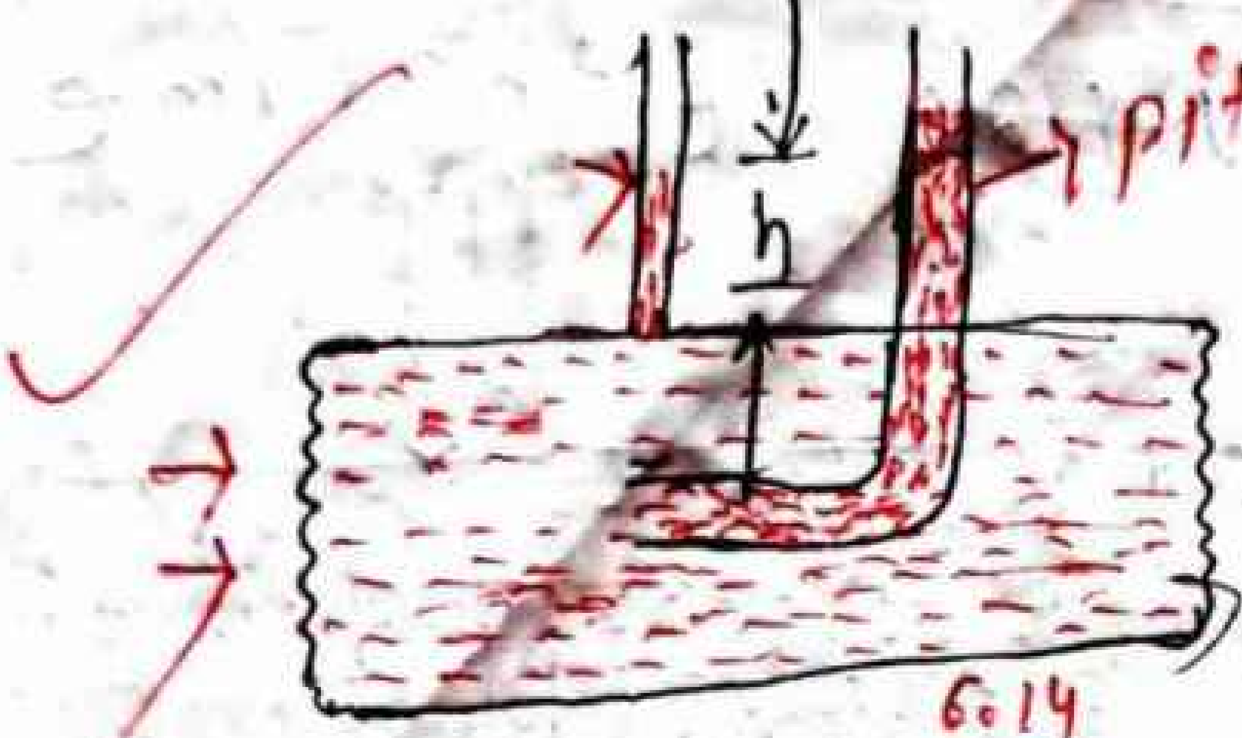
$$\boxed{v = C_v\sqrt{2gh}}$$

velocity of flow in a pipe by pitot - tube:— For finding the velocity at any point in a pipe by pitot tube, the following arrangement are adopted.

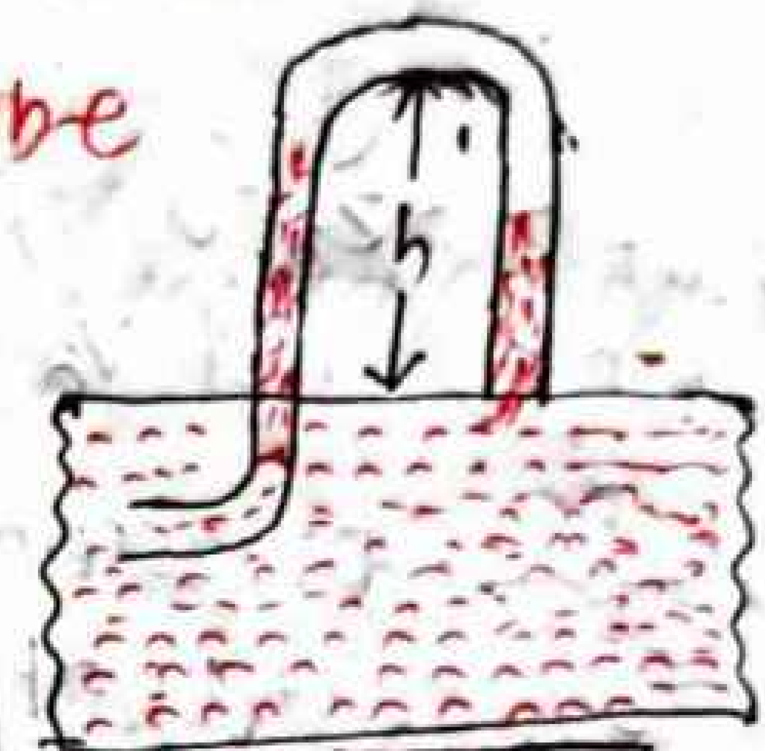
1- pitot tube along with a vertical piezometer tube as shown fig. 6.14

2 - pitot tube connected with piezometer tube as shown fig. 6.15

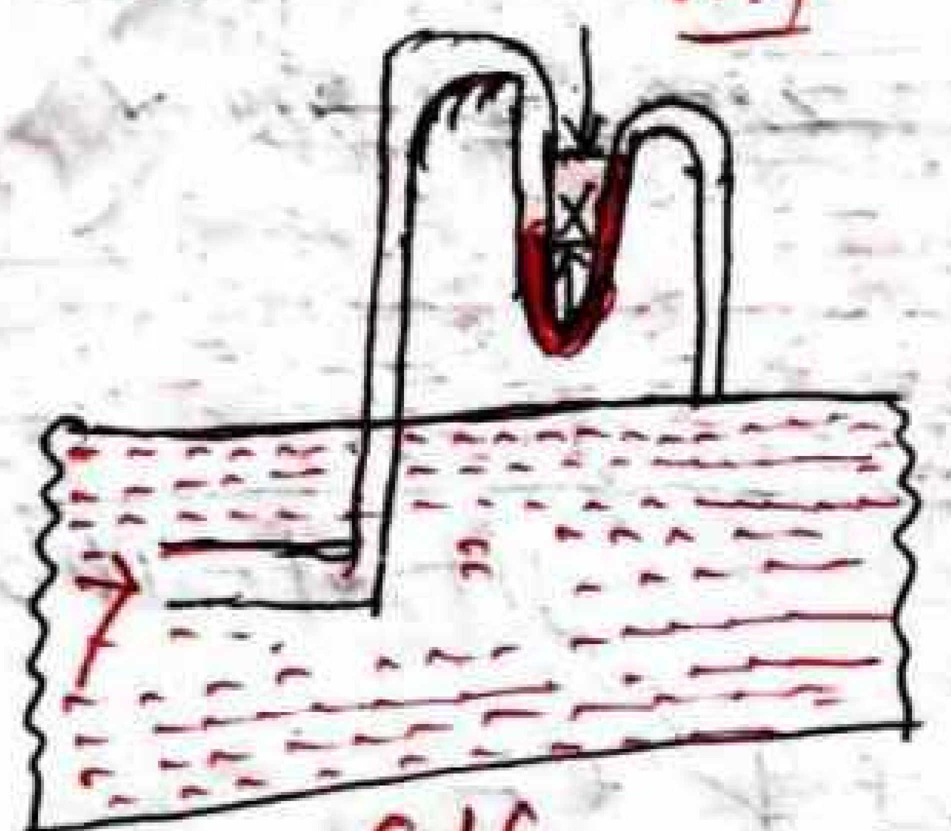
3 - pitot tube vertical piezometer tube connected with a different U-tube manometer as shown fig. 6.16



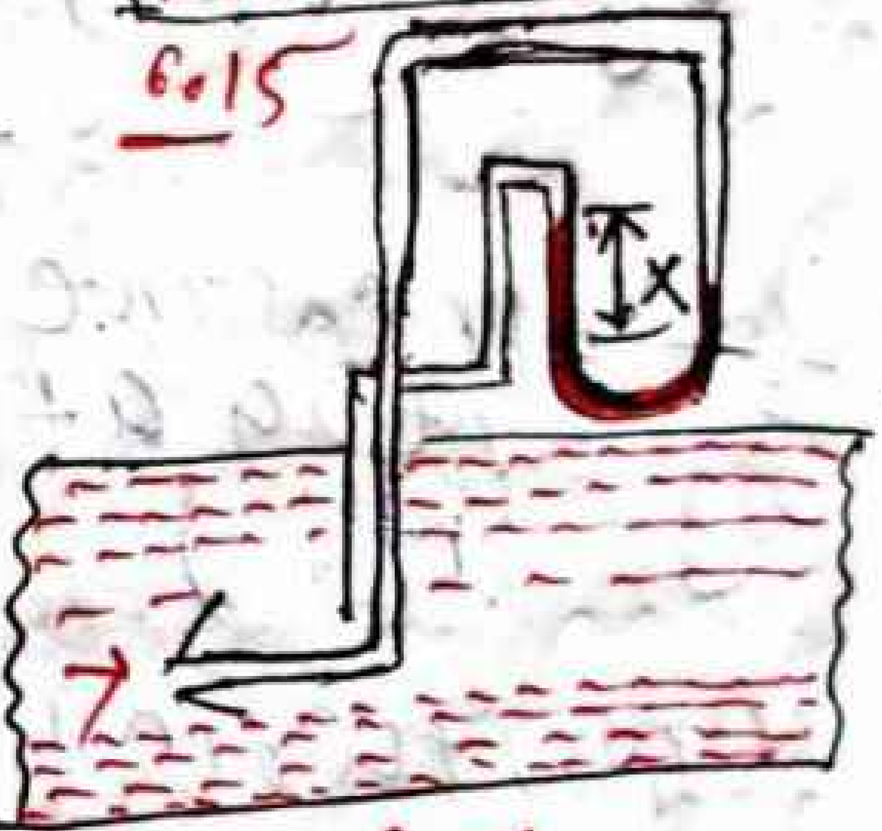
6.14



6.15



6.16



6.17

4- The pitot-static tube, which consist of circular concentric tube one inside the other with some annular space in between as show. 6.17

The outlet of these two tubes are connected to the differential manometer where the diffⁿ of pressure head 'h' is measured by knowing the diffⁿ of the levels of the manometer liquid say x.

$$\text{Then } h = x\left[\frac{S_g}{S_v} - 1\right]$$

Reynold's Number:

11/09/2023

- It is defined as the ratio of inertia force of the flowing fluid and the viscous force of the fluid.
- It is a dimensionless Number.
- It is denoted as "Re"

Mathematically:-

$$Re = \frac{\text{Inertia force}}{\text{viscous force}}$$

$$\Rightarrow Re = \frac{\rho v d}{\mu}$$ where:-

ρ = Density of the fluid
v = velocity of fluid
d = diameter of pipe
μ = viscosity of fluid

- $Re < 2000 \rightarrow Re > 4000 \rightarrow 2000 < Re < 4000 \rightarrow$

lamellar flow — Turbulant flow — Transitional flow

* Transitional flow is mixture of lamminar flow and Turbulant flow.
* Turbulant flow is done center.
* lamellar flow is done Age.

Ex:- stream or rivers into lake.

:- LOSS OF ENERGY IN PIPES :-

when a fluid is flowing through a pipe, the fluid experience some resistance due to which some of the energy of fluid is lost. This loss energy is classified as:-

Loss energy

major energy loss

This is due to friction and it is calculated by the following formulae

ⓐ Darcy-weisbach formula
ⓑ Chezy's formula

minor energy loss

Imp

This is due to
ⓐ sudden expansion of pipe
ⓑ sudden contraction of pipe
ⓒ Bend in pipe
ⓓ pipe fitting etc
ⓔ An obstruction in pipe.

Expression for loss of Head Due to friction in pipe :-

Consider a uniform horizontal pipe having steady flow in fig. 10.3

Let 1-1 and 2-2 are two section of pipe. Let

p_1 = pressure intensity at section-1-1
v_1 = velocity of flow at section 1-1
d = diameter of pipe

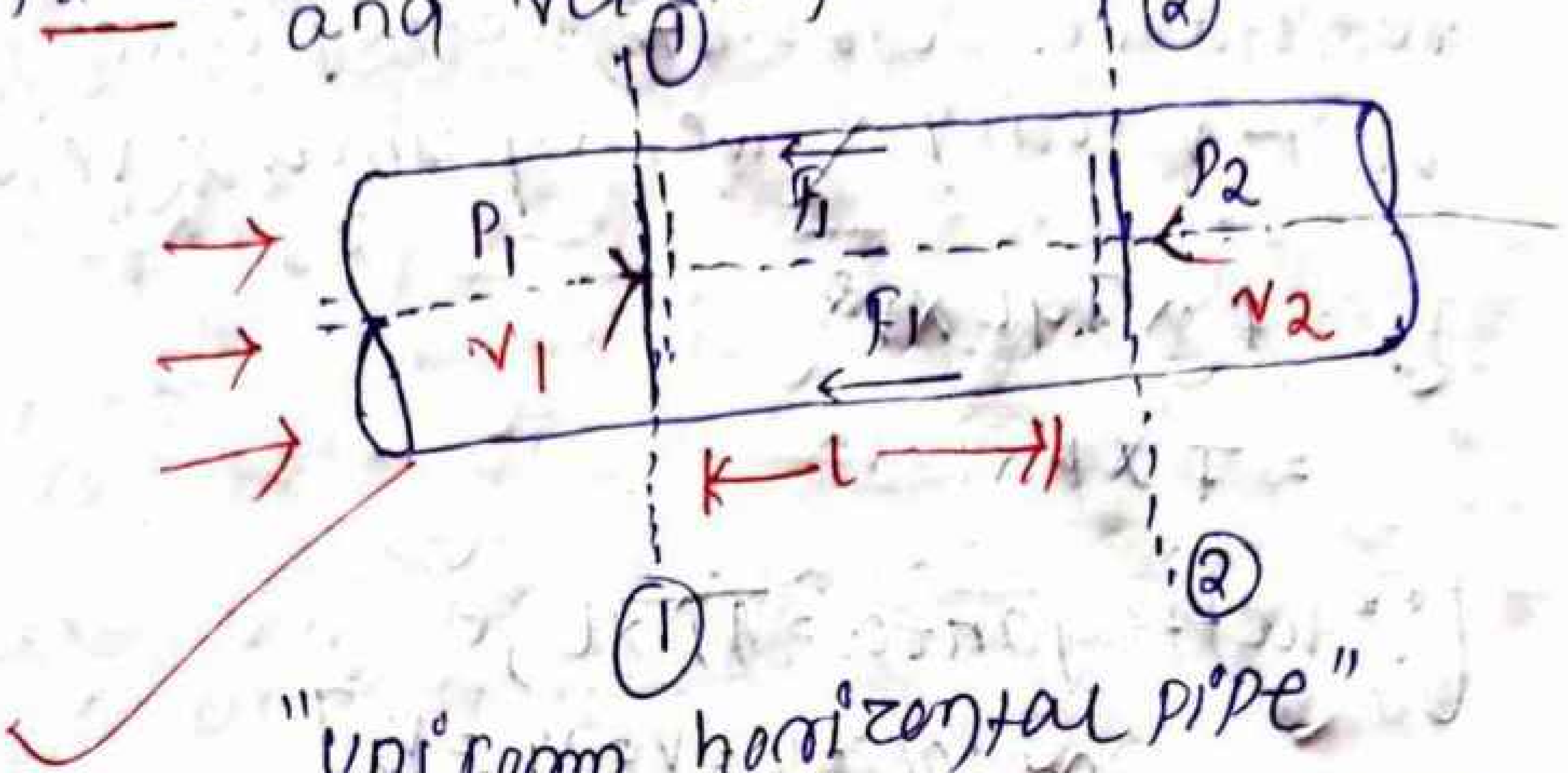
l = length of the pipe between 1-1 and 2-2

d = diameter of pipe

f' = frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction

& p_2, v_2 = are values of pressure intensity and velocity at section-2-2



"uniform horizontal pipe"

Applying Bernoulli's equation between section 1-1 and 2-2

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2

or $\dfrac{p_1}{\rho g} + \dfrac{v_1^2}{2g} + z_1 = \dfrac{p_2}{\rho g} + \dfrac{v_2^2}{2g} + z_2 + h_f$

But $z_1 = z_2$ as pipe is horizontal

$v_1 = v_2$ as dia. of pipe is same at (1-1) and (2-2)

$\therefore \dfrac{p_1}{\rho g} = \dfrac{p_2}{\rho g} + h_f$

or $h_f = \dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g}$ —— (1)

But h_f is the head lost due to friction and hence intensity of pressure will be reduce in the direction of flow by frictional resistance.

Now frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity²

$$F_1 = f' \times \pi d L \times V^2$$

$$= f' \times P \times L \times V^2$$

[∵ wetted area $= \pi D \times L$,
velocity $= V = V_1 = V_2$]

[∵ πD = perimeter $= P$ --- (ii)]

The forces acting on the fluid betn section 1-1 and 2-2 are

1. pressure force at sec. 1-1 $= P_1 \times A$
where A = Area of pipe
2. pressure force at sec 2-2 $= P_2 \times A$
3. Frictional force F_1 as shown in fig.

Resolving all forces in the horizontal direction, we have

$$P_1 A - P_2 A - F_1 = 0$$

or $(P_1 - P_2) A = F_1 = f' \times P \times L \times V^2$

or $P_1 - P_2 = \dfrac{f' \times P \times L \times V^2}{A}$

But from equation (i) $P_1 - P_2 = \rho g h_f$

$\left[\because \text{from (ii) } q_1 = f' p l v^2\right]$

equating the value of $(P_1 - P_2)$ we get

$$\rho g h_f = \frac{f' \times P \times L \times v^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times v^2 \quad \text{(iii)}$$

In eq^n (iii) $\frac{P}{A} = \frac{\text{wetted perimeters}}{\text{Area}}$

$$= \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$$

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times v^2 = \frac{f'}{\rho g} \times \frac{4 L v^2}{d} \quad \text{(iv)}$$

putting $\boxed{\frac{f'}{\rho} = \frac{f}{2}}$, where f is know as co-efficient of friction.

eq^n (iv) becomes as:-

$$h_f = \frac{4 \cdot f}{2g} \cdot \frac{L v^2}{d} = \frac{4 f \cdot L \cdot v^2}{d \times 2g} \quad \text{(v)}$$

eq^n (v) is know as Darcy-weisbach eq^n. This eq^n is commonly used for finding loss of head due to friction in the pipe.

some time eq^n (v) is written as

$$h_f = \frac{f \cdot L \cdot v^2}{d \times 2g} \quad \text{(v)}$$

Then f' is know as friction factor.

$v_1 = v_2$ | energy balance bet^n (1) & (2)
$z_1 = z_2$ |

~~Total energy at (1-1)~~

Total energy at (1-1) = Total energy at (2-2) + Head loss due to (hf)

$$\frac{p_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} + z_1 = \frac{p_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + z_2 + h_f$$

$$\Rightarrow \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = h_f$$

$$\Rightarrow \boxed{\frac{p_1 - p_2}{\rho g} = h_f} \text{ or } p_1 - p_2 = h_f \rho g$$

$F_1 = f' \times \pi D L \times V^2$

where

F_1 = Total frictional resistance

$\pi D L$ = Area of pipe

f' = frictional resistance per unit wetted area per unit velocity.

πD = P (perimeter of the cross section)

$$\boxed{F_1 = f' \times P L \times V^2}$$

Balance in Horizontal forces :-

$\Rightarrow P_1A - P_2A$ or $A(P_1 - P_2) = 0$

$\Rightarrow P_1A - P_2A - F_1 = 0$

$\Rightarrow F_1 = (P_1 - P_2)A$

$\Rightarrow \dfrac{f' \times P \times L \times V^2}{A} = (P_1 - P_2)$

$\Rightarrow \dfrac{f' \times P \times L \times V^2}{A} = \rho g h_f$

$\Rightarrow h_f = \dfrac{f' \times P \times L \times V^2}{\rho g A}$

$\Rightarrow h_f = \dfrac{f'}{\rho g} \times \dfrac{P}{A} \times L \times V^2$ (1)

where

$\dfrac{P}{A}$ wetted perimeter

$$\boxed{= \frac{\pi}{\frac{\pi}{4} d^2} \times d = \frac{4}{d}}$$

$= \dfrac{f'}{\rho} \times \dfrac{4}{dg} \times L V^2$

where

$$\boxed{\frac{f'}{\rho} = f/2}$$ → Imperical formula

f = friction factor

$$h_f = \frac{f}{2} \times \frac{4}{dg} \times l v^2$$

$$\Rightarrow \boxed{h_f = \frac{4flv^2}{2dg}}$$

This equation is called Darcy weisbach equation.

Head loss: 2/9/2023

* Head loss is P.E i.e converted to K.E

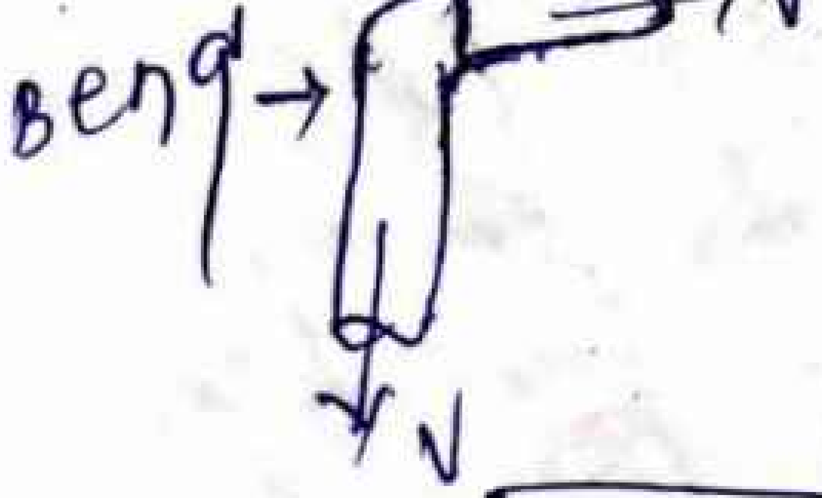
where

P.E = potential energy

K.E = Kinetics energy

i.e : That is

① **minor head loss due to pipe Bend:**



{pipe bend is used to change the direction of flow}

$$\boxed{h_{bend} \text{ or } h_b = \frac{k_b v^2}{2g}}$$

where
Kb = co-efficient of bend
v = velocity of fluid
g = constant = 9.81 m/s^2

hb = loss of head due to bend.

* The value of Kb depends on angle of bend radius of curvature of bend and diameter of pipe.

*

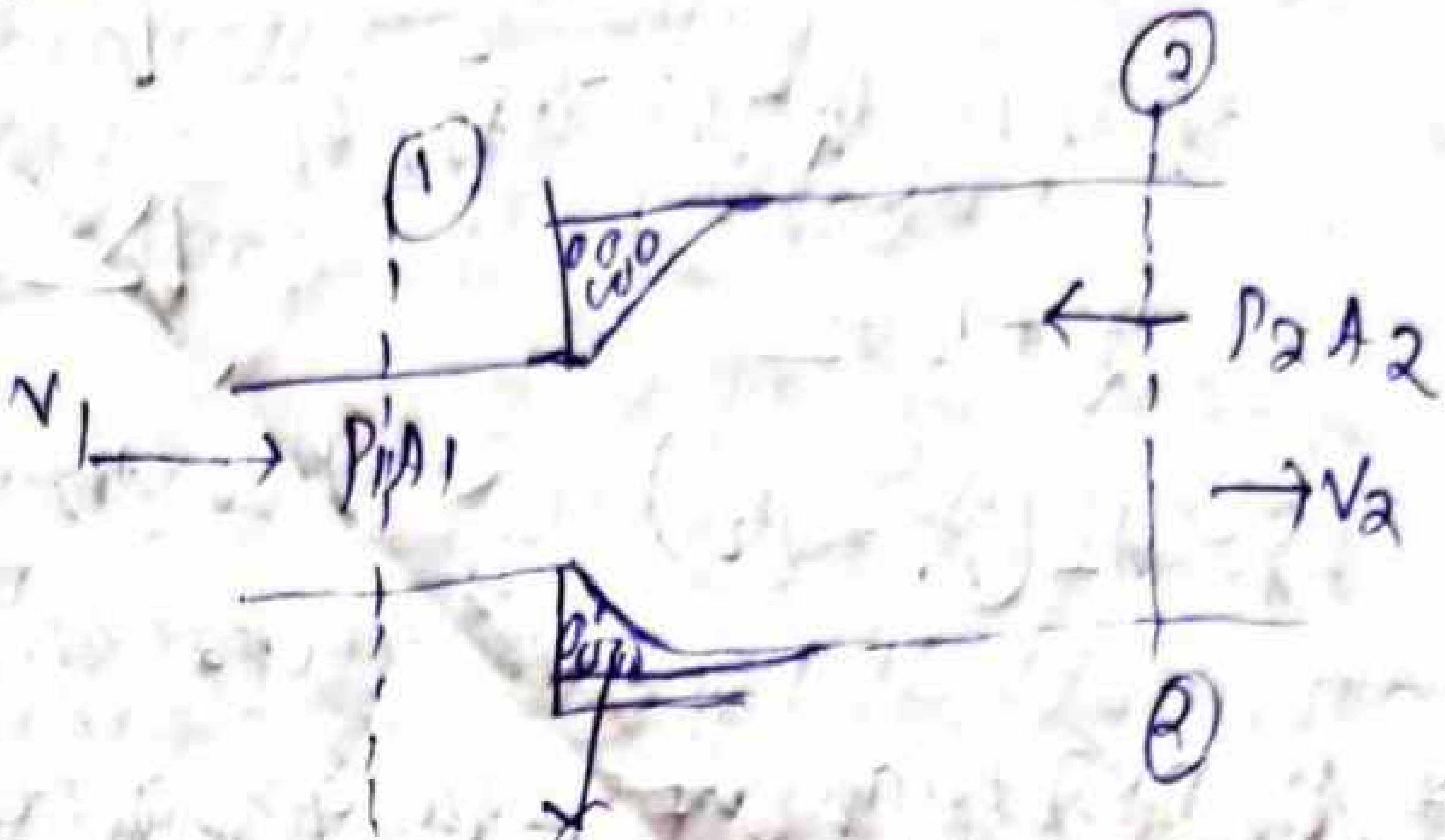
The minor Energy (Head) losses :-

* The loss of head energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy include the following case :-

① loss of head due to sudden enlargement.
② loss of head due to sudden contraction.
③ loss of head at the entrance of a pipe.
④ loss of head at the exit of a pipe.
⑤ loss of head due to an obstruction in a pipe.
⑥ loss of Head due to bend in the pipe.
⑦ loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe these losses are comparable with the loss of head due to friction.

② Loss of Head due to sudden enlargement :—



① Eddy current

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_e \qquad (z_1 = z_2)$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_e \quad \left[\text{head loss enlargement}\right]$$

$$\Rightarrow h_e = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{v_1^2 - v_2^2}{2g}\right) \quad \text{—} \otimes$$

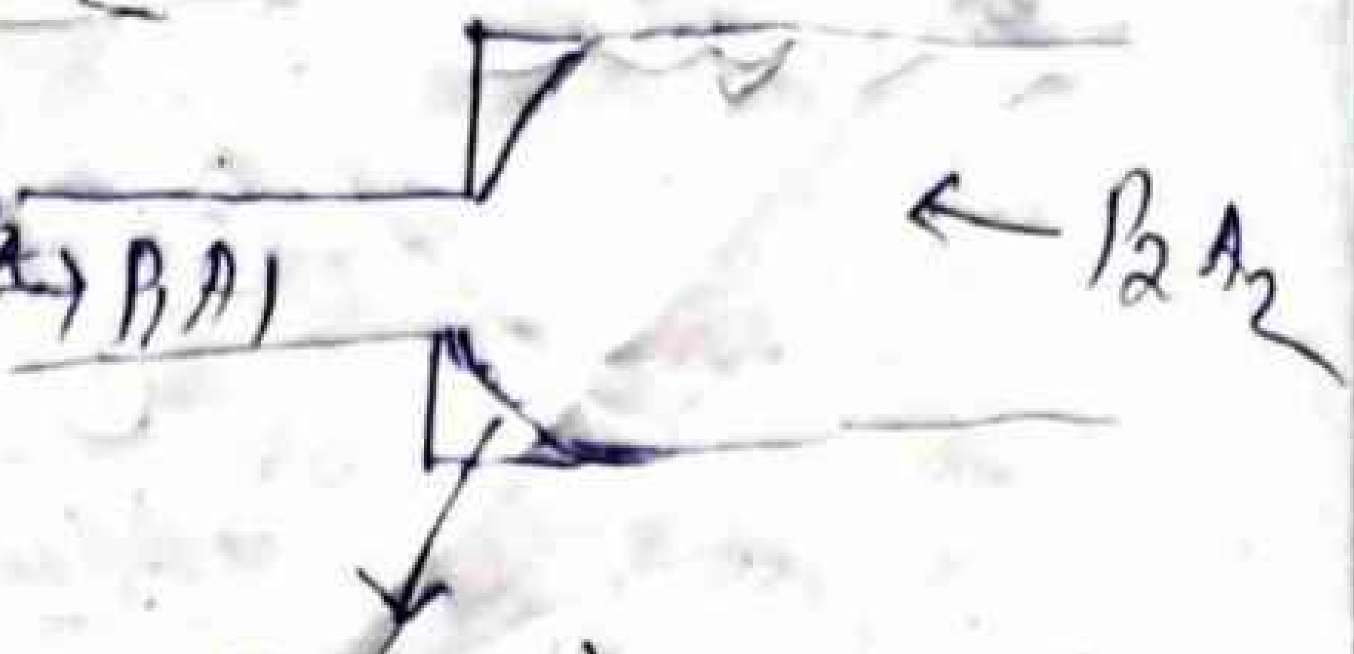
$$\Rightarrow h_e = \frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g} \quad \text{—} \ (1)$$

* net force in the direction of fluid flow

$$F_x = P_1 A_2 + P(A_2 - A_1) - P_2 A_2$$

$F_x = P_1A_1 + P(A_2 - A_1) - P_2A_2$

$= P_1A_1 + P_1A_2 - \cancel{P_1A_1} - P_2A_2$



$= P_1A_2 + P_2A_2$

$P(A_2 - A_1)$

$F_x = A_2(P_1 - P_2)$

$\Rightarrow P_1(A_2 - A_1)$

* momentum of liquid or second at section (1)

$(\rho A_1 v_1) v_1$ or $\rho A_1 v_1^2$ — sec (1)

* momentum of liquid or second at section 2 & 2

$\rho (A_2 v_2) v_2$

$= \boxed{\rho A_2 v_2^2}$ — sec (2)

change in momentum = final - initial

$\Rightarrow (P_1 - P_2) A_2 = \rho A_2 v_2^2 - \rho A_1 v_1^2$

$\Rightarrow (P_1 - P_2) A_2 = \rho (A_2 v_2^2 - A_1 v_1^2)$

Applying continuity equation

$A_1 v_1 = A_2 v_2$

$\Rightarrow A_1 = \frac{A_2 v_2}{v_1}$

$\boxed{A_1 = \frac{A_2 v_2}{v_1}}$

$\Rightarrow (P_1 - P_2) A_2 = \rho (A_2 v_2^2 - A_1 v_1^2)$

$\Rightarrow (P_1 - P_2) A_2 = \rho \left(A_2 v_2^2 - \frac{A_2 v_2}{v_1} \cdot v_1^2\right)$

$\Rightarrow (P_1 - P_2) A_2 = \rho (A_2 v_2^2 - A_2 v_2 \cdot v_1)$

$\Rightarrow (P_1 - P_2) \cancel{A_2} = \rho \cancel{A_2} (v_2^2 - v_2 \cdot v_1)$

$\Rightarrow (P_1 - P_2) = \rho (v_2^2 - v_2 \cdot v_1)$

$\Rightarrow \frac{P_1 - P_2}{\rho} = v_2^2 - v_1 v_2$

Divin ding g of on both side

$$\frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g} \quad \text{——— (ii)}$$

putting the value of $\left(\frac{P_1 - P_2}{\rho g}\right)$ in eqn (1)

$$h_e = \frac{P_1 - P_2}{\rho g} + \frac{v_1^2 - v_2^2}{2g}$$

$$h_e = \frac{v_2^2 - v_1 v_2}{g} + \frac{v_1^2 - v_2^2}{2g}$$

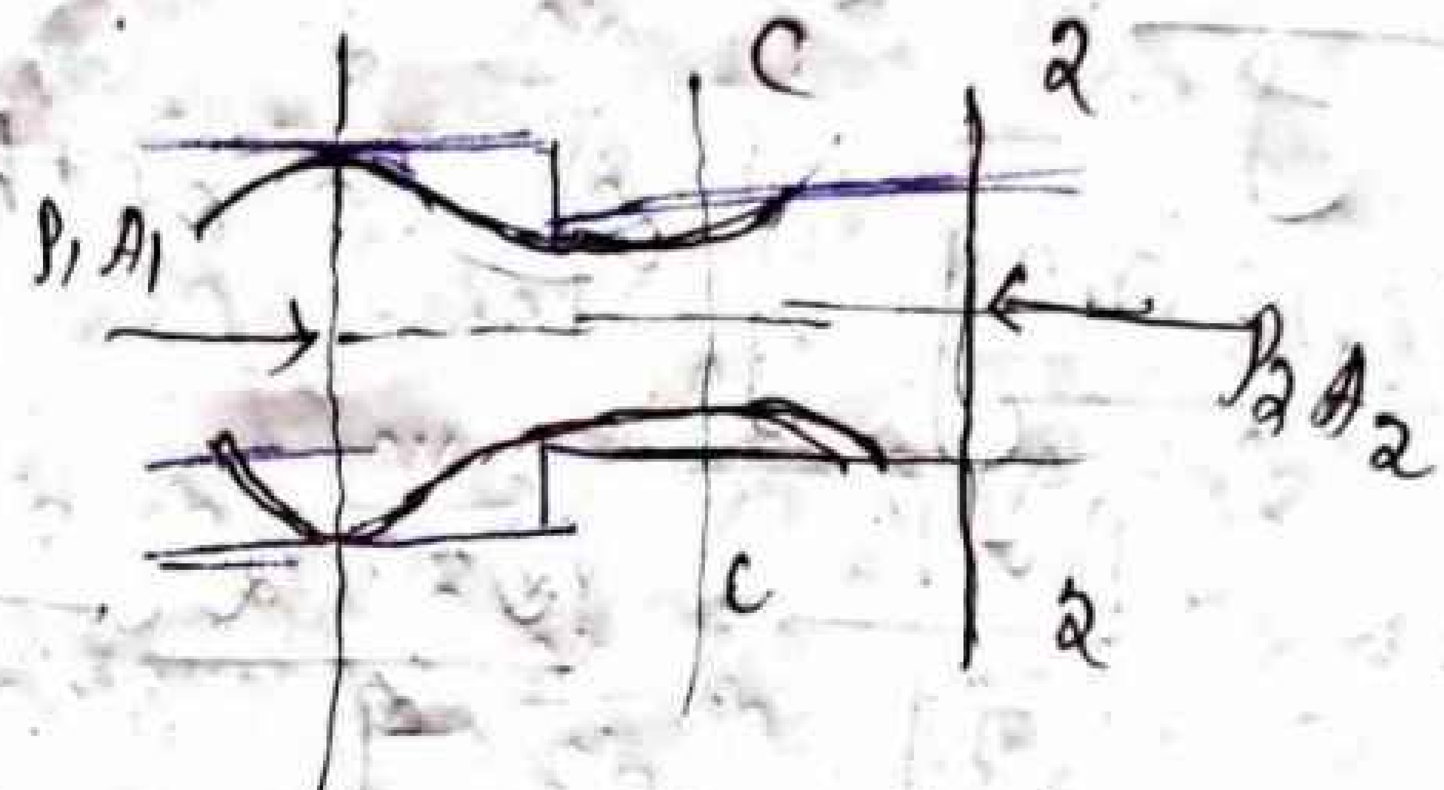
$$h_e = \frac{2v_2^2 - 2v_1 v_2 + v_1^2 - v_2^2}{2g}$$

$$h_e = \frac{v_2^2 - 2v_1 v_2 + v_1^2}{2g} = \frac{(v_1 - v_2)^2}{2g}$$

The final equation of head loss enlargement is

$$\boxed{he = \frac{(v_1 - v_2)^2}{2g}}$$

loss of head due to sudden contraction:-



section (c-c) is called vena-contraction

sec (1,1) to se (x,x) flow is reduce sec (x,x) to sec (c,c) flow is further reduce, sec (c,c) to sec (2,2) flow sudden enlargement

Here we can only considers between sec 1 to sec 2,2

→ The head loss

$$he = \frac{(v_c - v_2)^2}{2g}$$

$$= v_2^2 \left[\frac{v_c}{v_2} - 1\right]^2 \quad \text{—— (1)}$$

From continuity equation

$$A_C V_C = A_2 V_2$$

$$\Rightarrow \frac{V_C}{V_2} = \frac{A_2}{A_C} = \frac{1}{A_C/A_2} = \frac{1}{C_e},$$

substituting the value of

$$\therefore = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]$$

$$h_c = \frac{kV_2^2}{2g} \quad \text{where} \left[\frac{1}{C_e} - 1\right]^2 = k$$

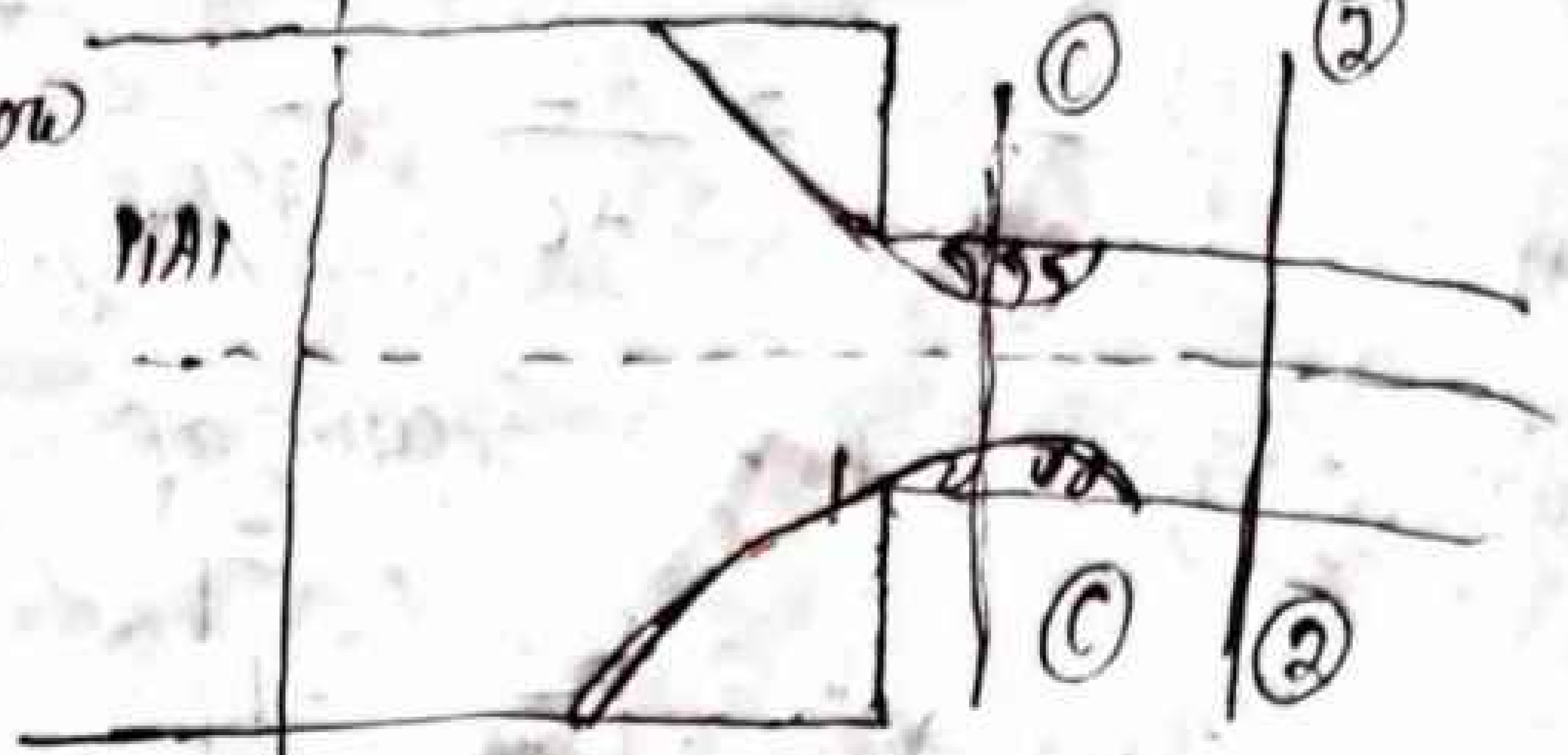
If the value of C_e is assumed 0.62 then $k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$

Then $h_c = \frac{kV_2^2}{2g} = 0.375 \frac{V_2^2}{2g}$.

Loss of Head due to sudden contraction :-

consider a liquid flowing in a pipe which has a sudden contraction in area ~~as shown fig.~~ consider two sec 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe. the area of flow goes on decreasing and becomes minimum at a section C-C ~~as shown fig.~~ 11.2 This sec C-C is called vena-contraction. After sec C-C a sudden enlargement of the area take place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena contraction to smaller pipe

Let A_c = Area of flow at section c-c
v_c = velocity of flow at sec. c-c
A_2 = Area of flow at sec. 2-2
V_2 = velocity of flow at sec 2-2.
h_c = loss of head due to sudden contraction.



Now h_c = actual loss of head due to enlargement from sec. c-c to sec 2-2 and given by eqn.

$$\frac{(v_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{v_c}{V_2} - 1\right]^2 \quad \text{—①}$$

from continuity eqn we have

$$A_c v_c = A_2 V_2 \text{ or } \frac{v_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c}$$

substituting the value of $\frac{v_c}{V_2}$ in eqn(1) we get $\therefore \left[C_c = \frac{A_c}{A_2}\right]$

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 \quad \text{—— (ii)}$$

$$= \frac{kV_2^2}{2g}, \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62 then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Then, h_c become as $h_c = \frac{kV_2^2}{2g} = 0.375\frac{V_2^2}{2g}$

If the value of C_c is not given then the head loss due to contraction is taken as $\boxed{0.5\frac{V_2^2}{2g} \text{ or } h_c = \frac{V_2^2}{2g}\, 0.5}$

Loss of Head due to Bend in pipe :-

When there is any bend in a pipe, the velocity of flow changes due to which the separation of the flow from the boundary and also formation of eddies take place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as.

$$h_b = \frac{kV^2}{2g}$$

where

h_b = Loss of head due to bend

v = velocity of flow

k = co-efficient of bend

The value of k depends on

1. Angle of bend
2. Radius of curvature of bend
3. Diameter of pipe

unit-2 4/9/2023

Fourier's law of Heat conduction

$$Q \propto A \cdot \frac{dt}{dx}$$

where

Q = Heat flow through a body per unit time

A = surface area of Heat flow

dt = Temp. diff of the faces of block of thickness - d

dx = Through which heat flow dx is thickness of body in the direction of flow.

$$\Rightarrow Q = -KA\frac{dt}{dx}$$

k = Thermal conductivity (const. proportionality) also know as Thermal conductivity of body.

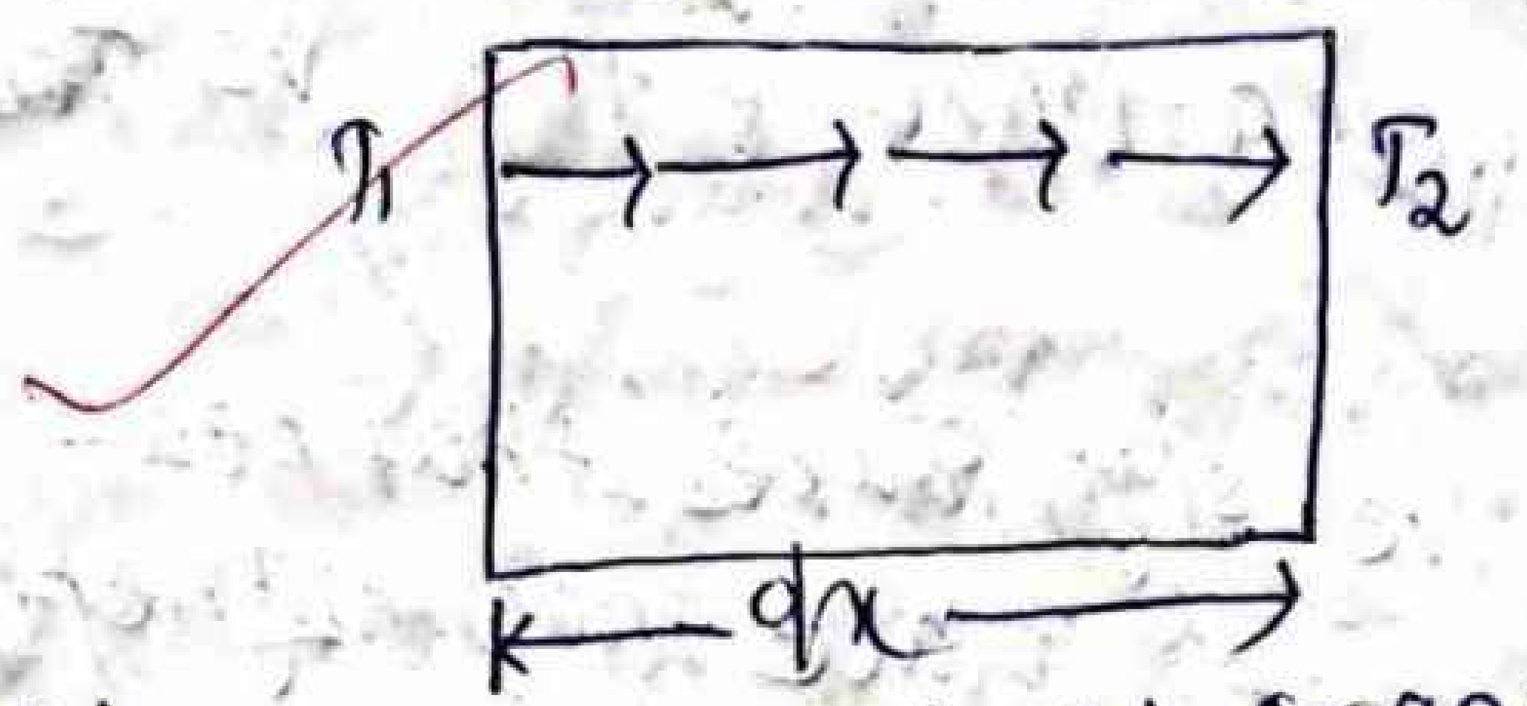
'-' is indicated by Temperature difference

Fourier's law of conduction of heat:-

The law state that the rate of heat transfer flux is directly proportional to the -ve temperature graidient.

Derivation:-

Generally in conduction process rate of heat transfer is directly proportional to the temperature difference between hot and cold point and also directly proportional to the direction of flow but inversly proportional to the distance between hot and cold point.



let Q = Rate of heat transfer (speed)

$dt = t_1 - t_2$ = Temperature difference between hot and cold point.

dx = Thickness of material.

A = Area of the material through which heat exchange occurs.

According to fourier's law $Q \propto A(t_1-t_2) \times \frac{1}{dx}$

firstly $Q \propto A$ —① (Area)

$Q \propto (t_1 - t_2)$ —② (Temp. diff.)

$Q \propto \frac{1}{dx}$ —③ (distance)

from eqn ①, ② and ③ we get the fourier's law as $Q \propto \frac{A(t_2-t_1)}{dx} \Rightarrow Q = \frac{-kA(t_2-t_1)}{dx}$

Where

K : Thermal conductivity of the medium

$\Rightarrow \frac{Q}{A} = -K\left(\frac{t_2 - t_1}{dx}\right)$

Here Q/A represent rate of heat flow.

$\Rightarrow q = \frac{-K(t_2 - t_1)}{dx}$

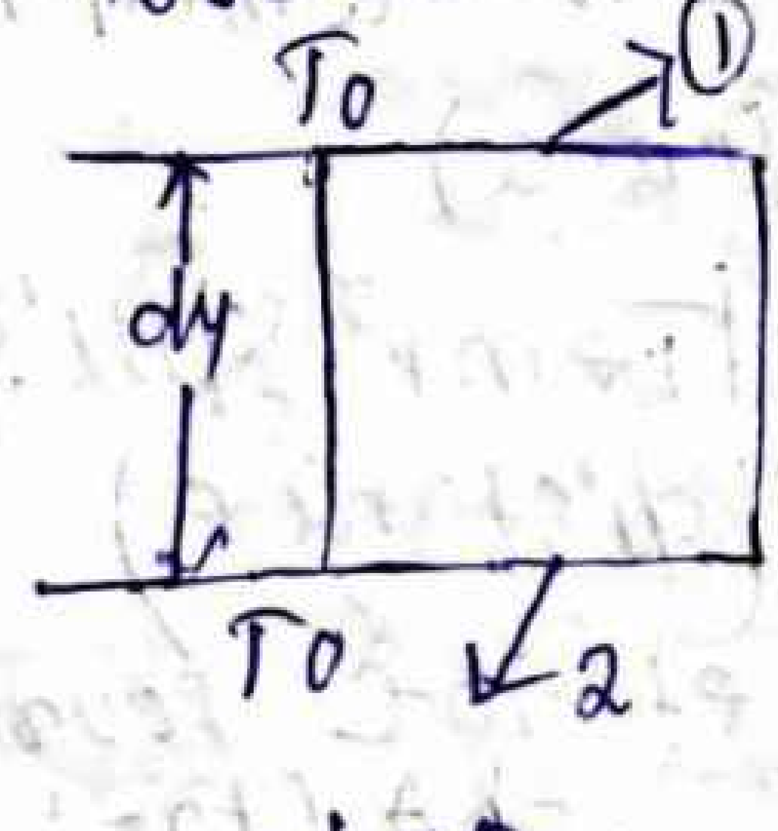
$\Rightarrow q \propto -\frac{dt}{dx}$

This show that heat flow is directly proportional to the -ve Tempr. gradient.

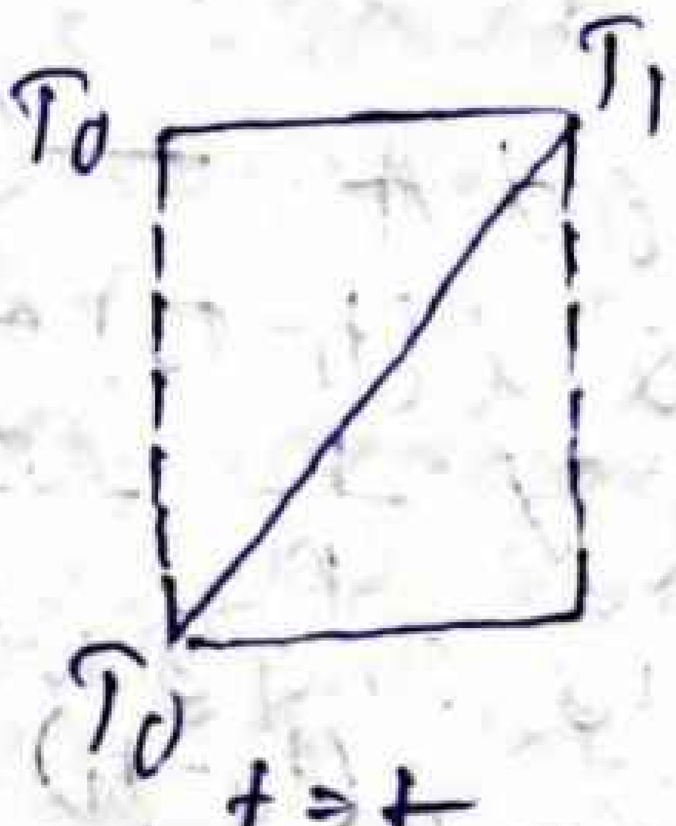
The negative sign makes that heat transfer takes place from higher Tempr. to lower Tempr.

It also indicate that the temperature decreases with increasing thickness of material.

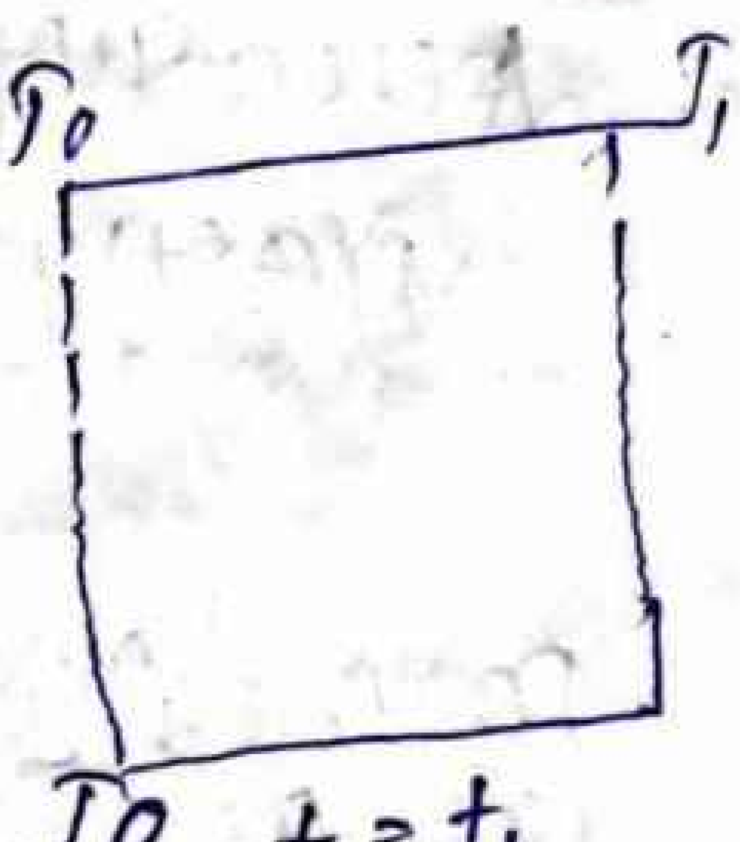
Derivation of Fourier's law from 1st principle :-



t = 0
Fig - 1



t = t
Fig - 2



t = t_1
Fig - 3

① consider two plate ① and ② initially in figure-1, both the surface are at temperature

~~(T₀) hence the upper~~

T_0 hence there is no temperature gradient the temperature disturbution is an show in the figure-1.

② suddenly the upper surface is heated to a temperature (T_1) while the lower is at (T_1), t = at the tempr distribution over "dy" is a shown in fig-2.

③ $t = t_1$, where sufficient time has been allowed steady state heat flow condition is reached where a constant amount of heat flow between the surface ① and ② under such condition the tempr. profile dy becomes is straight line.

at is found ($T_1 - T_0$) is small qy is proportional to ($T_1 - T_0$) and inversly proportional to the distance 'y' between the surface.

$$qy \propto T_1 - T_0$$

$$\propto 1/y$$

$$qy = \frac{k(T_1 - T_0)}{y}$$

This relationship is fourier's law under differential condition in three dimensional heat flow.

$$qy = -k \frac{dT}{dy}$$

$$q_x = -k \frac{dT}{dx}$$

$$q_z = -k \frac{dT}{dz}$$

So that heat flow in three dimensional is

$$Q = q_x + q_y + q_z$$

$$Q = -k\left(\frac{dT}{dx} + \frac{dT}{dy} + \frac{dT}{dz}\right)$$

* units of q

$$q = cal/sec.cm^2$$

* unit of Q

$$Q = q.A \ cal/sec.$$

Thermal conductivity of MATERIALS:—

From eq^n (1.1) $k = \frac{Q}{A} \cdot \frac{dx}{dt}$

Then value of $k = 1$ when $Q = 1$, $A = 1$ and $\frac{dt}{dx} = 1$

Now $k = \frac{Q}{1} \cdot \frac{dx}{dt}$ (unit of k: $W \times \frac{1}{m^2} \times \frac{m}{K(°C)}$

$= W/mK$. or $W/m°C$)

Thus the Thermal conductivity of a material is defined as follows.

→ The amount of energy conducted through a body of unit area and unit Thickness in unit time when the difference in temperature between the face causing heat flow is unit Temp^r diff.

→ It follows from eq^n (1.1) the material with high thermal conductivities are good conductor of heat, whereas materials with low thermal conductivities are good thermal insulator.

→ conduction of heat occurs most readily in pure metal, less in alloys and much less readily in non-metal.

→ The very low thermal conductivities of certain insulator e.g cork is due to their porosity, the air trapped within the material acting as an insulator.

→ Thermal conductivity (a property of material) depends essentially upon the following factor

1. material structure
2. moisture content
3. Density of the material
4. pressure and tempr (operating condition)

THERMAL RESISTANCE: (Rth)

→ when two physical system are described by simillar eqn and have simillar boundary cond, these are said to be analogus.

→ The heat transfer process may be compared by analogy with the flow of electricity in an electrical resistance.

→ As the flow of electric current in the electrical resistance is directly proportional to potential diffn (dV).

→ similarly heat flow rate Q, is directly proportional to tempr. diffn (dt) the driving force for heat conduction through a medium.

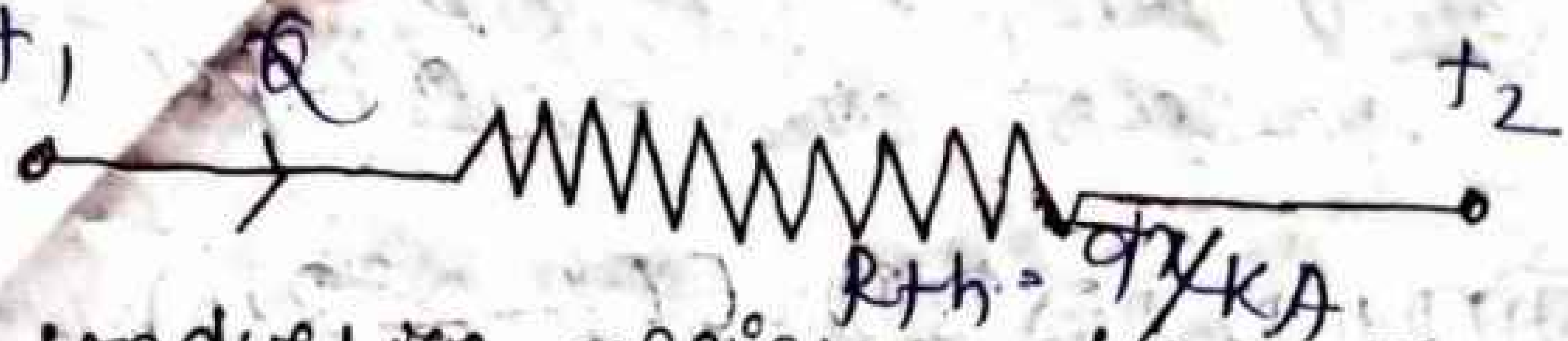
As per ohm's law (in electric-circuit theory), we have

$$\text{current}(I) = \frac{\text{potential difference}(dV)}{\text{Electrical resistance}(R)}$$

By analogy the heat flow eq^n (fouriers eq^n) may be written as

$$\text{Heat flow rate}(Q) = \frac{\text{Temp}^r \text{ difference } dt}{\frac{dx}{KA}}$$

By comparing eq^n (1.4) and (1.5), we find that 'I' is analogous to Q. dV is analogous to dt and 'R' is analogous to the quantity $\frac{dx}{KA}$, the quantity $\frac{dx}{KA}$ is called



Thermal conduction resistance $(Rth)_{cond}$ i.e

$$(Rth)_{cond} = \frac{dx}{KA}$$

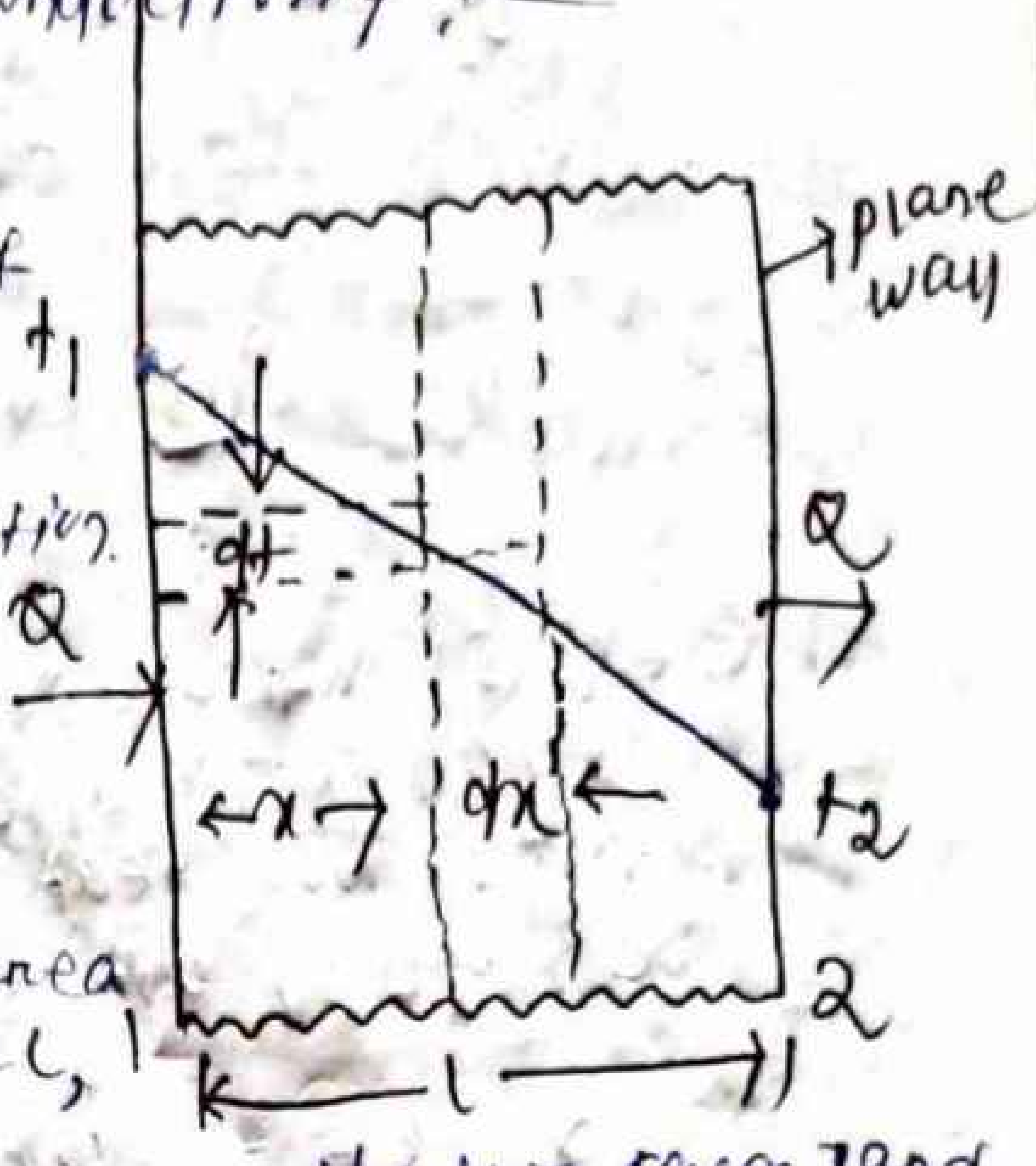
- The reciprocal of the thermal resistance is called thermal conductance.
- It may be noted that rule for combining electrical resistance in series and parallel apply equally well to thermal resistance.

The concept of Thermal resistance is quite helpful while making calculation for flow of heat

HEAT CONDUCTION THROUGH A PLANE WALL :-

Case-1 Uniform Thermal conductivity :-

Refer to fig.



(a) consider a plane wall of homogeneous material through which heat is flowing only in - x - direction.

let.

l = Thickness of the plane wall.

A = cross-sectional area of the wall material, and

t_1, t_2 = Tempr maintained at the two faces 1 and 2 of the wall respectively.

$(R_{th})_{cond} = \frac{l}{kA}$

(Heat conduction Through a plane wall)

The general heat conduction eqn in cartesian co-ordinates is given by

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \quad \text{—— (I)}$$

If the heat conduction take place under the condition steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, one dimensional $\left[\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0\right]$ and with no internal heat generation $\left(\frac{q_g}{k} = 0\right)$ then the above eqn is reduced to

$$\frac{\partial^2 t}{\partial x^2} = 0, \text{ or } \frac{d^2 t}{dx^2} = 0 \quad \text{—— (II)}$$

By integrating the above differential twice we have,

$$\frac{dt}{dx} = C_1 \text{ and } t = C_1 x + C_2 \quad \text{—— (III)}$$

where C_1 and C_2 are the arbitary constant. The values of these constant may be calculated from the known boundary condition as follows.

At $x = 0$ $\quad t = t_1$

$x = l$ $\quad t = t_2$

substituting the value in the eq^n(3) we get

$$t_1 = 0 + C_2 \quad \text{and} \quad t_2 = C_1 l + C_2$$

After simplification, we have $C_2 = t_1$ and

$$C_1 = \frac{t_2 - t_1}{l}$$

Thus the eq^n(3) reduce to:

$$t = \left(\frac{t_2 - t_1}{l}\right) x + t_1 \quad \text{—— (4)}$$

The eq^n(5) indicates that Temp^r distribution across a wall is linear and is dependent of thermal conductivity. Now heat through the plane wall can be found by using Fourier's eq^n as follows:

$$Q = -kA \frac{dt}{dx} \quad \left[\text{where } \frac{dt}{dx} = \text{Temp}^r \text{ gradient}\right]$$

$$\frac{dt}{dx} = \frac{d}{dx}\left[\left(\frac{t_2 - t_1}{l}\right) x + t_1\right] = \frac{t_2 - t_1}{l}$$

$$Q = -kA \frac{(t_2 - t_1)}{l} = \frac{kA(t_1 - t_2)}{l} \quad \text{—— (5)}$$

Eq^n(5) can be written as:

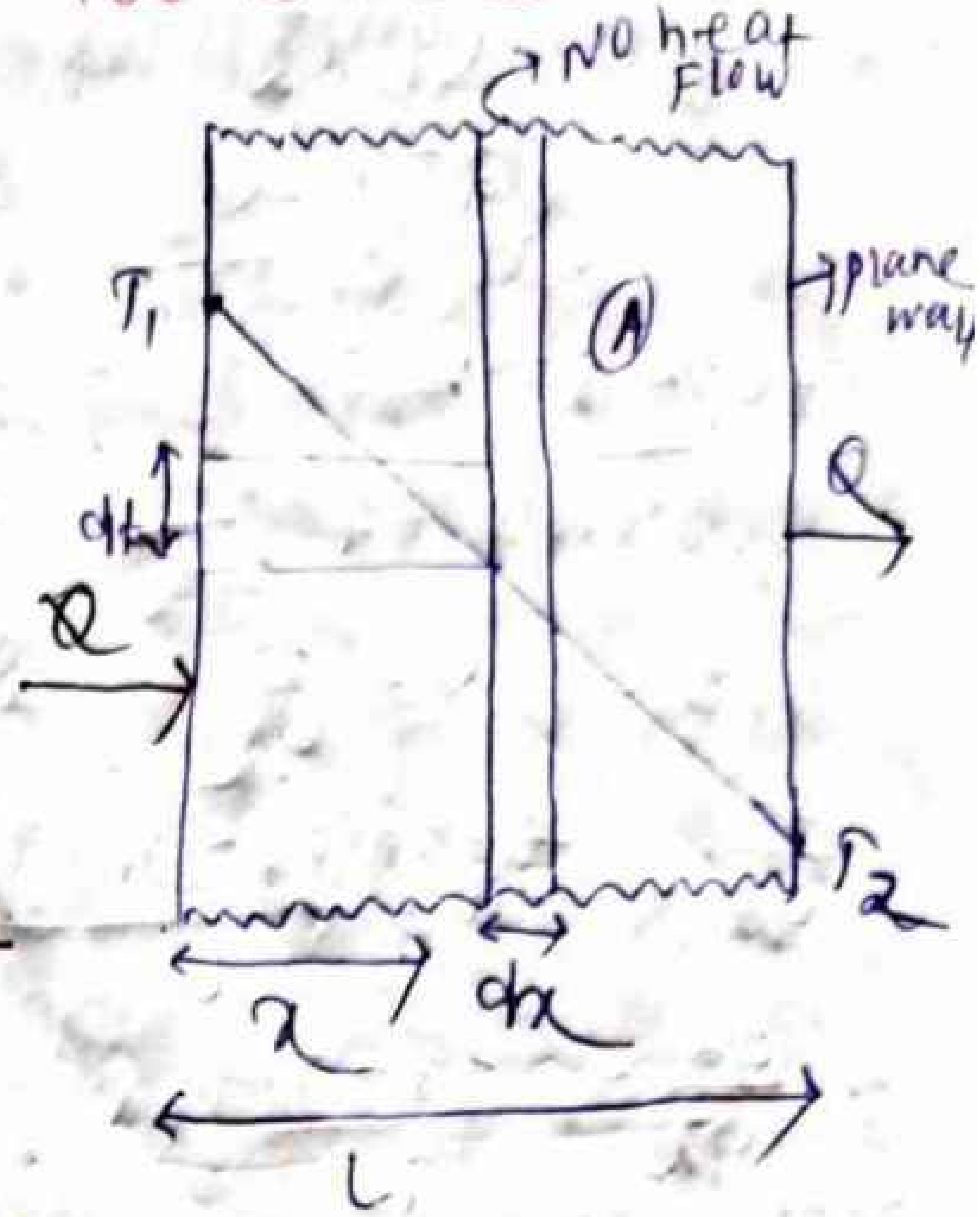
$$Q = \frac{(t_1 - t_2)}{(l/kA)} = \frac{t_1 - t_2}{(R_{th})_{cond.}} \quad \text{—— (6)}$$

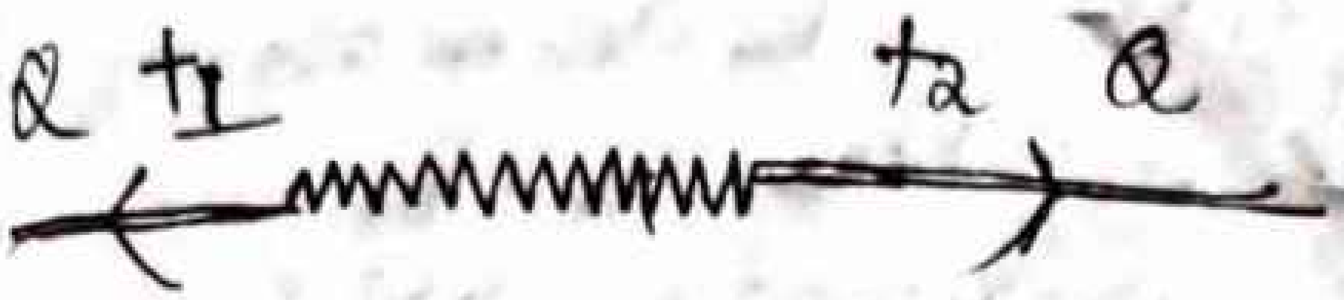
where,

$(R_{th})_{cond}$ = Thermal resistance to heat conduction. fig. show the equivalent thermal circuit for heat flow through the plane wall.

Steady state conduction to plane wall :—

A = crossectional area
K = Thermal conductivity
T_1 = Initial Tempr.
T_2 = Final Tempr.
l = Thickness of the wall.





Step-1

Starting with general heat eqn cartesian co-ordinates.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{qg}{K} = \frac{1}{a}\frac{\partial t}{\partial z}$$

Step-2

(a) steady state $\frac{\partial t}{\partial \tau} = 0$

(b) one dimensional $\frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial z^2} = 0$

(c) No internal heat generate $\frac{qg}{K} = 0$

$$\frac{\partial^2 t}{\partial x^2} = 0 \quad \frac{\partial^2 t}{\partial x^2} = 0$$

Step-3 :—

$$\int \frac{\partial^2 t}{\partial x^2} = 0 \quad \int \frac{dt}{dx} = C_1$$

$$\Rightarrow \frac{t}{x} = C_1 + C_2$$

$$\Rightarrow t = C_1 x + C_2$$

Step-4 :—

At $t = t_1$, $x = 0$

$t = t_2$, $x = L$ [Total length]

$t = C_1 x + C_2$ —(1)

$\Rightarrow t_1 = C_1 \times 0 + C_2$

$\Rightarrow \boxed{t_1 = C_2}$

Then

$t_2 = C_1 L + t_1$

$$\boxed{C_1 = \frac{t_2 - t_1}{L}}$$

$t = C_1 x + C_2$

$t = \frac{(t_2 - t_1)x}{L}$

Step-5 :-

Fourier's eqn :-

$$Q = -kA \frac{dt}{dx}$$

$$\frac{dt}{dx} = \frac{d}{dx}\left[\frac{(t_2 - t_1)x}{L} + t_1\right]$$

$$\Rightarrow \frac{dt}{dx} = \frac{t_2 - t_1}{L}$$

$$\Rightarrow Q = -kA\left(\frac{t_2 - t_1}{L}\right) \Rightarrow Q = k\left(\frac{t_1 - t_2}{L}\right)$$

Step-6 :- Find the thermal resistance

$$Q = \frac{kA}{L}(t_1 - t_2)$$

$$R_t = \frac{t_1 - t_2}{Q}$$

putting the value of 'Q' we get

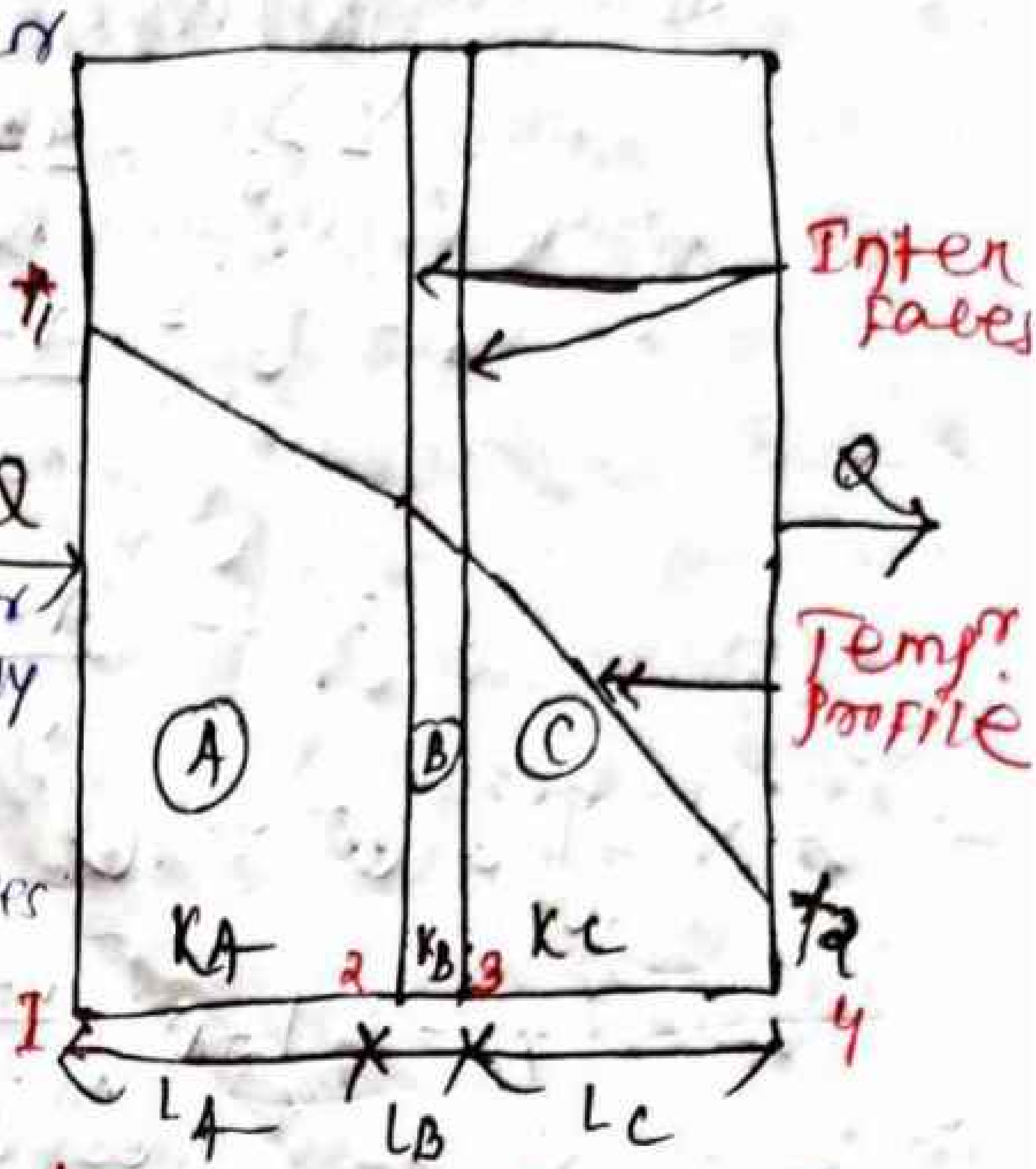
$$R_t = \frac{t_1 - t_2}{\frac{kA}{L}(t_1 - t_2)}$$

$$\boxed{R_t = \frac{L}{kA}}$$

R_t = Thermal resistance of the wall

Heat conduction through a composite wall:-

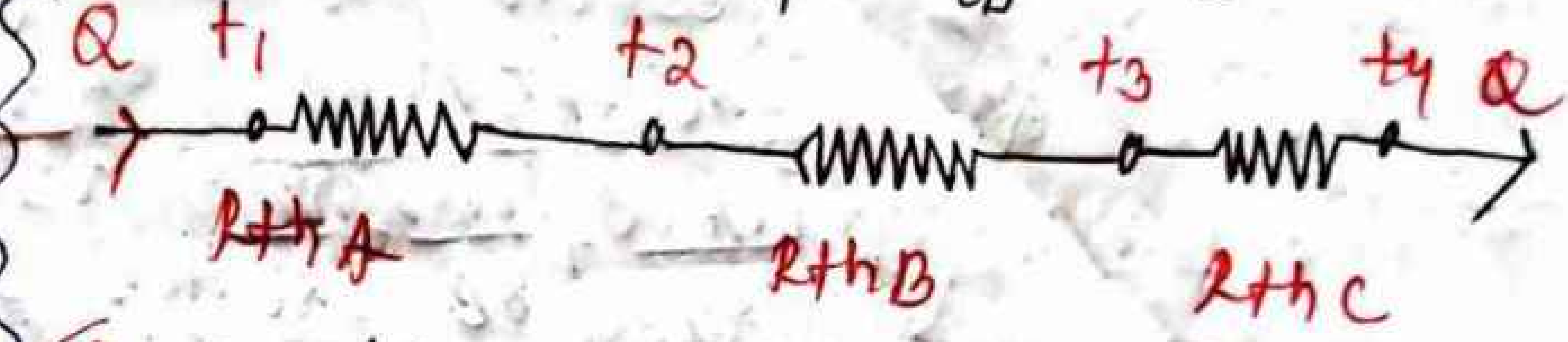
Refer to Fig.(2.6) consider the transmission of heat through a composite wall consisting of a number of slabs.



Let

$l_A\ l_B\ l_C$ = Thickness of slabs or wall A, B, & C respectively (also called path-length)

$k_A\ k_B\ k_C$ = Thermal conductivities of the wall A, B & C respectively.



$$R_{thA} = \frac{l_A}{k_A \cdot A} \quad R_{thB} = \frac{l_B}{k_B \cdot A} \quad R_{thC} = \frac{l_C}{k_C \cdot A}$$

Steady state conduction through a composite wall.

$T_1\ t_4\ (t_1 > T_4)$ = Tempr at the wall surface 1 & 4 respectively, and,

$T_2\ T_3$ = Tempr at the interface 2' and 3 respectively.

Since the quantity of heat transmitted per unit time through each slab/layer is same we have.

$$Q = \frac{k_A \cdot A(t_1 - t_2)}{l_A} = \frac{k_B \cdot A(t_2 - t_3)}{l_B} = \frac{k_C \cdot A(t_3 - t_4)}{l_C}$$

Assuming that there is a perfect contact betn the layer and no Tempr drop occurs across the interface between materials

$$R_{thA} = \frac{L_A}{k_A \cdot A},\ R_{thB} = \frac{L_B}{k_B \cdot A},\ R_{thC} = \frac{L_C}{k_C \cdot A}$$

Steady state conduction through a composite wall.

Rearranging the above expression we get

$$t_1 - t_2 = \frac{Q \cdot l_A}{k_A \cdot A} \quad \text{--- (I)}$$

$$t_2 - t_3 = \frac{Q \cdot l_B}{k_B \cdot A} \quad \text{--- (II)}$$

$$t_3 - t_4 = \frac{Q \cdot l_C}{k_C \cdot A} \quad \text{--- (III)}$$

Adding (i) (ii) and (iii) we have

$$(t_1 - t_4) = Q\left[\frac{l_A}{k_A \cdot A} + \frac{l_B}{k_B \cdot A} + \frac{l_C}{k_C \cdot A}\right]$$

$$Q = \frac{A(t_1 - t_4)}{\left[\frac{l_A}{k_A} + \frac{l_B}{k_B} + \frac{l_C}{k_C}\right]} \quad \text{--- (IV)}$$

$$Q = \frac{(t_1 - t_4)}{\left[\frac{l_A}{k_A \cdot A} + \frac{l_B}{k_B \cdot A} + \frac{l_C}{k_C \cdot A}\right]}$$

$$= \frac{t_1 - t_4}{\left[R_{th-A} + R_{th-B} + R_{th-C}\right]}$$

If the composite wall consist of 'n' slabs/layers then

$$Q = \frac{\left[t_1 - t_{(n+1)}\right]}{\sum_{1}^{n} \frac{l}{k_A}}$$

10. CONVECTION:— 13/09/23

⇒ It is a process in which heat flow is achieved by actual motion of warmer portions with cooler portion of same material.

⇒ It's the heat transfer due to bulk movement within fluid such as gases and liquids.

⇒ For example:- heating of water by hot surface is mainly by convection.

Convection is of two type

(1) Natural convection (Free convection)

(2) Forced convection (Artificial convection)

NATURAL CONVECTION:—

⇒ It refers to a case where the fluid movement is created by the warm fluid it self.

⇒ The density of fluid decrease as it is heated. Thus, hot fluids are lighter than cool fluid.

continue - - - - - - - ?

* Natural convection is defined as heat transfer convection process in which a fluid is heated, the currents set up may cause mixing of fluid.

* This type of heat transport, in which fluid motion is not generated by external source (like pump, fan, suction device etc.)

* In natural convection, fluid surrounding heat source receives heat and by thermal expansion become less dense and rises.
* The surrounding, cooler fluid is then moves to replace it.
* This cooler fluid is then heated and the process continues, forming convection current.

Cont.
* This process transfers heat energy from the bottom of the convection cell to top.
* The driving force for natural convection is buoyancy, a result of difference in fluid density.
* This process continues thereby effecting the mixing of hot and cold fluids.

$q \propto (T_s - T_\infty)$ [$\because q$ = heat]

$q = h(T_s - T_\infty)$

h = heat transfer co-efficient for convection heat flow

Application :-
* Natural convection is observed when extracts are evaporated in open pans.

Forced Convection :-
→ Forced convection is a heat transfer convection process in which mixing of fluid may be obtained by the use of stirrer or agitator or pumping the fluid or recirculation.
→ Consider a case of heat flowing from a hot fluid through a metal wall into cold fluid.

→ variation of temp^n at a specific point is observed.

$$Q = hA(T_s - T_\infty)$$

Q: Heat transfer

h = heat transfer co-efficient of forced convection.

A: Area m^2

T_∞ = Temp^n of fluid at distance well removed from the surface (hence the stagant fluid temp^n)

Natural convection :-

$q \propto (T_s - T_\infty)$ [∵ q = heat]

$q = h(T_s - T_\infty)$

h = heat transfer co-efficient for convection heat flow.

Diff^n betn natural vs forced convection :-

key points.	Natural convection	forced convection
Definition	• A method of heat transfer in which the motion of fluid is influenced by natural means.	• A method of heat transfer in which the motion of fluid is influenced by external means.
Initiation of fluid motion	• fluid motion generates as a result of the change of the density of fluid when heat.	• fluid motion generate as a result of an external source such as pumping, fan suction devices.
Factor affecting Heat transfer	• No external factor affected the heat transfer.	• only external factor can cause heat transfer.
Other names.	• The other name of natural convection is free convection.	• The other name of forced convection is Artificial convection.
Examples	• Cooling down a boiled egg when kept in the normal air, loss of eff! of a cool drink can.	• Air conditioning system, Turbine etc.