THEORY OF STRUCTURES(TH -2)

QUESTION BANK

4TH SEMESTER(CIVIL ENGINEERING)

(As per SCTE &VT new syllabus)

UNIT-I

Direct and Bending Stresses in Vertical Members

A) 2 Marks Questions (with Answers)

1. Define axial load and eccentric load.

Answer: Axial load acts along centroidal axis producing uniform stress. Eccentric load acts at a distance (eccentricity) from centroid producing direct + bending stresses.

2. State the formula for direct stress.

Answer: Direct stress $\sigma d = P/A$, where P = load and A = cross-sectional area.

3. State the formula for bending stress.

Answer: Bending stress $\sigma b = My/I$, where M = bending moment, y = distance from neutral axis, I = moment of inertia.

4. Write expression for maximum and minimum stress under eccentric load.

Answer: $\sigma max = P/A + My/I$ and $\sigma min = P/A - My/I$.

5. What is middle third rule?

Answer: For rectangular section, if load lies within middle third, no tension develops on the section.

6. Define core of section.

Answer: Core is the area within which load must act so that stresses remain compressive throughout.

7. What is limit of eccentricity for rectangular section?

Answer: $e \le b/6$ (about axis parallel to breadth) and $e \le d/6$ (about axis parallel to depth).

8. What is limit of eccentricity for circular section?

Answer: $e \le D/8$ or $e \le r/4$.

9. State condition for zero stress at extreme fibre.

Answer: $\sigma min = 0 \Rightarrow P/A = My/I$.

10. Why chimneys are checked for wind pressure?

Answer: Wind causes bending moment and tensile stress which may crack masonry, hence stress check is essential.

B) 5 Marks Questions (with Solutions)

1. Derive expressions for maximum and minimum stresses in a rectangular section subjected to eccentric load about one axis.

Solution:

1. Given rectangular section

Let

- Breadth = **b**
- Depth = **d**
- Area,

$$A = bd$$

A **compressive load** Pacts with **eccentricity** $e$about **one axis** (say about the **centroidal y–y axis**, i.e. eccentricity along depth).

2. Nature of loading

Because the load is eccentric, it produces:

1. **Direct stress**
2. **Bending stress**

So this is a case of **combined direct stress + bending stress**.

3. Direct stress due to load *P*

$$\sigma_d = \frac{P}{A} = \frac{P}{bd}$$

This stress is **uniform** over the entire section.

4. Bending moment due to eccentricity

Eccentric load produces a bending moment:

$$M = P \times e$$

5. Moment of inertia of rectangular section

About centroidal axis perpendicular to depth:

$$I = \frac{bd^3}{12}$$

6. Bending stress at any fibre

$$\sigma_b = \frac{My}{I}$$

Maximum bending stress occurs at extreme fibre:

$$y = \frac{d}{2}$$

$$\sigma_b = \frac{Pe(\frac{d}{2})}{\frac{bd^3}{12}}$$

Simplifying:

$$\sigma_b = \frac{6Pe}{bd^2}$$

7. Combined stress at extreme fibres

Total stress = **Direct stress ± Bending stress**

(a) Maximum stress

Occurs at fibre **nearest to load**:

$$\sigma_{\max} = \frac{P}{bd} + \frac{6Pe}{bd^2}$$

(b) Minimum stress

Occurs at fibre **farthest from load**:

$$\sigma_{\min} = \frac{P}{bd} - \frac{6Pe}{bd^2}$$

8. Final expressions (boxed form)

$$\boxed{\sigma_{\max} = \frac{P}{bd}(1 + \frac{6e}{d})}$$

$$\boxed{\sigma_{\min} = \frac{P}{bd}(1 - \frac{6e}{d})}$$

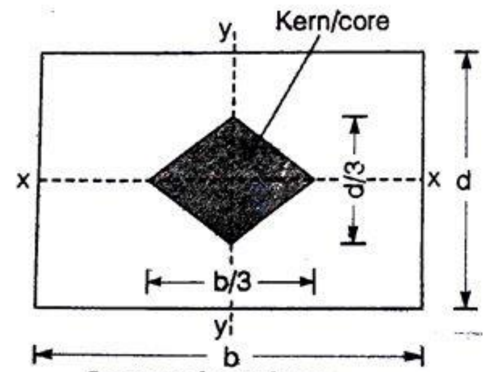
2.Explain Middle third rule of rectangular section with sketch .

Middle Third Rule (Rectangular Section)

When a **rectangular section** is subjected to an **eccentric compressive load**, stresses developed at the base depend on the **position of the load**.
The **Middle Third Rule** states the condition under which **no tensile stress** occurs in the section.

For a rectangular section under eccentric compression, if the line of action of the load lies within the middle one-third of the base, the stress over the entire section remains compressive (no tension occurs).

.Rectangular Section Details



Rectangular Column

Let,

- Breadth of section = **b**
- Depth of section = **d**
- Area, $A = b \times d$
- Load = **P**
- Eccentricity about centroidal axis = **e**

. Stress Due to Eccentric Load

An eccentric load causes:

1. **Direct compressive stress**

$$\sigma_d = \frac{P}{A}$$

Bending stress

$$\sigma_b = \frac{My}{I}$$

where

$$M = Pe$$

. Maximum and Minimum Stress

For rectangular section,

$$I = \frac{bd^3}{12}$$
$$y = \frac{d}{2}$$

So,

$$\sigma_{max} = \frac{P}{A} + \frac{Pe \cdot (d/2)}{I}$$
$$\sigma_{min} = \frac{P}{A} - \frac{Pe \cdot (d/2)}{I}$$

. Condition for No Tension

For **no tensile stress**,

$$\sigma_{min} \geq 0$$

Substitute:

$$\frac{P}{A} - \frac{Pe(d/2)}{I} \geq 0$$

Divide by P:

$$\frac{1}{A} \geq \frac{e(d/2)}{I}$$

Substitute $A = bd$and $I = \frac{bd^3}{12}$:

$$\frac{1}{bd} \geq \frac{e(d/2)}{(bd^3/12)}$$

Simplifying,

$$e \leq \frac{d}{6}$$

. Middle Third Rule Result

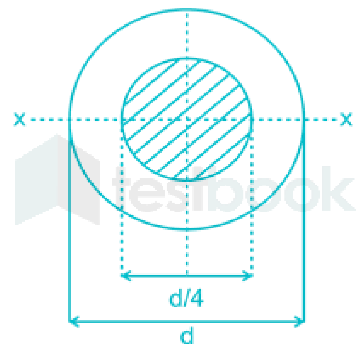
$$\boxed{e \leq \frac{d}{6}}$$

This means the load must lie within:

$$\boxed{\text{Middle one-third of the depth}}$$

3. Explain core of circular section and derive limit of eccentricity.

Solution:



An eccentric load causes:

(a) Direct Stress

$$\sigma_d = \frac{P}{A}$$

(b) Bending Stress

$$\sigma_b = \frac{My}{I}$$

where,

$$M = Pe$$

. Maximum and Minimum Stress

At extreme fibers:

$$\sigma_{max} = \frac{P}{A} + \frac{Pey}{I}$$
$$\sigma_{min} = \frac{P}{A} - \frac{Pey}{I}$$

For **no tension**:

$$\sigma_{min} \geq 0$$

2. Substitute Section Properties (Circular Section)

- Area:

$$A = \pi R^2$$

- Moment of inertia:

$$I = \frac{\pi R^4}{4}$$

- Extreme fiber distance:

$$y = R$$

3. Apply No-Tension Condition

$$\frac{P}{\pi R^2} - \frac{PeR}{(\pi R^4/4)} \geq 0$$

Cancel **P** and **π**:

$$\frac{1}{R^2} - \frac{4e}{R^3} \geq 0$$

Multiply by R^3:

$$R - 4e \geq 0$$

4. Limit of Eccentricity

$$\boxed{e \leq \frac{R}{4}}$$

5. Core of Circular Section

Hence, the **core** of a circular section is:

$$\boxed{\text{A concentric circle of radius } \frac{R}{4}}$$

4. Calculate maximum and minimum stresses at base of chimney due to wind moment.

Solution:
Step 1: Nature of stresses at chimney base

At the base of the chimney, two stresses act:

1. **Direct compressive stress** due to self-weight
2. **Bending stress** due to wind moment

Hence, total stress =

$$\sigma = \sigma_d \pm \sigma_b$$

Step 2: Direct stress due to self-weight

$$\sigma_d = \frac{W}{A}$$

For a circular section:

$$A = \frac{\pi D^2}{4}$$

Step 3: Bending stress due to wind moment

Bending stress at any fiber is:

$$\sigma_b = \frac{My}{I}$$

Where

- $y = \frac{D}{2}$

For a circular section:

$$I = \frac{\pi D^4}{64}$$

Step 4: Maximum stress at base

Maximum stress occurs on the **windward side**, where bending stress adds to direct stress.

$$\boxed{\sigma_{\max} = \frac{W}{A} + \frac{My}{I}}$$

Substituting values:

$$\sigma_{\max} = \frac{W}{\frac{\pi D^2}{4}} + \frac{M(\frac{D}{2})}{\frac{\pi D^4}{64}}$$

Step 5: Minimum stress at base

Minimum stress occurs on the **leeward side**, where bending stress subtracts from direct stress.

$$\boxed{\sigma_{\min} = \frac{W}{A} - \frac{My}{I}}$$

$$\sigma_{\min} = \frac{W}{\frac{\pi D^2}{4}} - \frac{M(\frac{D}{2})}{\frac{\pi D^4}{64}}$$

Final Answer (boxed form)

$$\boxed{\sigma_{\max} = \frac{W}{A} + \frac{My}{I}}$$

$$\boxed{\sigma_{\min} = \frac{W}{A} - \frac{My}{I}}$$

5. Differentiate between axial load and eccentric load.

Solution:

Difference Between Axial Load and Eccentric Load

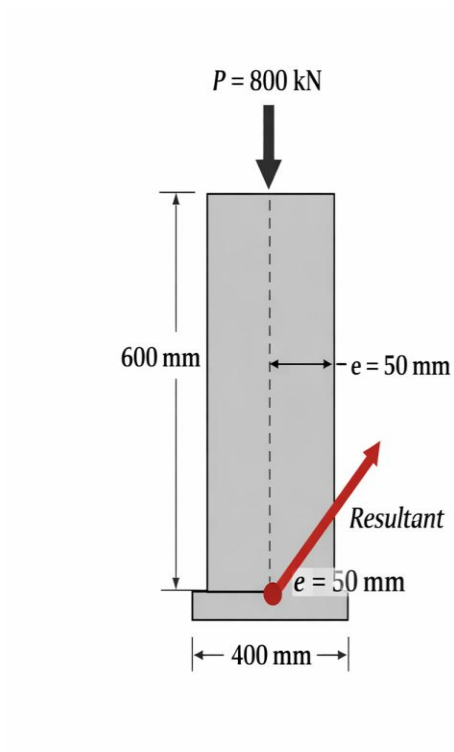
Basis	**Axial Load**	**Eccentric Load**
Definition	A load whose line of action passes through the centroid (axis) of the cross-section	A load whose line of action does **not** pass through the centroid of the cross-section
Eccentricity (e)	Eccentricity = 0	Eccentricity ≠ 0
Type of Stress Produced	Produces **only direct (uniform) stress**	Produces **direct stress + bending stress**
Stress Distribution	Stress is **uniform** over the entire cross-section	Stress is **non-uniform**; maximum at one edge and minimum at the opposite edge
Bending Moment	No bending moment is produced	Bending moment = Load × eccentricity
Possibility of Tension	No tension occurs if load is compressive	Tension may occur if eccentricity is large

Basis	Axial Load	Eccentric Load
Examples	Centrally loaded column, tie rod under direct pull	Chimney, retaining wall, column with offset load

C) 10 Marks Questions (with Solutions)

1. A rectangular pier 400 mm × 600 mm carries a load of 800 kN with eccentricity 50 mm about the 600 mm axis. Find maximum and minimum stresses.

Solution:



Step 1: Area of cross-section

$$A = b \times d = 400 \times 600 = 240000 \text{ mm}^2$$

Step 2: Direct stress due to axial load

$$\sigma_d = \frac{P}{A}$$
$$\sigma_d = \frac{800 \times 10^3}{240000}$$
$$\sigma_d = 3.33 \text{ N/mm}^2$$

Step 3: Bending moment due to eccentricity

$$M = P \times e$$
$$M = 800 \times 10^3 \times 50$$
$$M = 40 \times 10^6 \text{ N\cdotpmm}$$

Step 4: Moment of inertia about 600 mm axis

Since bending is **about the 600 mm axis**,

- depth $d = 400$ mm
- breadth $b = 600$ mm

$$I = \frac{bd^3}{12}$$

$$I = \frac{600 \times 400^3}{12}$$
$$I = \frac{600 \times 64 \times 10^6}{12}$$
$$I = 3.2 \times 10^9 \text{ mm}^4$$

Step 5: Distance of extreme fiber from neutral axis

$$y = \frac{d}{2} = \frac{400}{2} = 200 \text{ mm}$$

Step 6: Bending stress

$$\sigma_b = \frac{My}{I}$$
$$\sigma_b = \frac{40 \times 10^6 \times 200}{3.2 \times 10^9}$$
$$\sigma_b = 2.5 \text{ N/mm}^2$$

Step 7: Maximum and minimum stresses

(a) Maximum stress

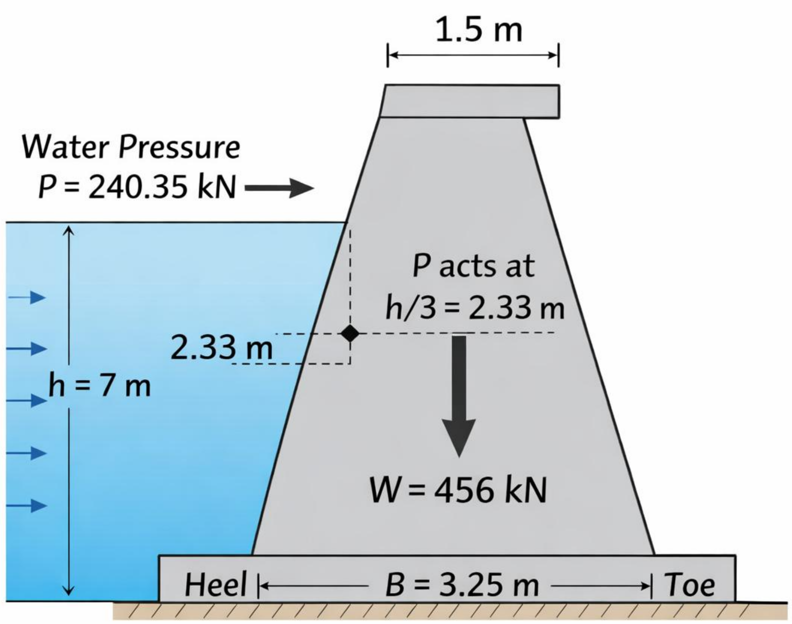
$$\sigma_{max} = \sigma_d + \sigma_b$$
$$\sigma_{max} = 3.33 + 2.5$$
$$\boxed{\sigma_{max} = 5.83 \text{ N/mm}^2 \text{ (compressive)}}$$

(b) Minimum stress

$$\sigma_{min} = \sigma_d - \sigma_b$$
$$\sigma_{min} = 3.33 - 2.5$$
$$\boxed{\sigma_{min} = 0.83 \text{ N/mm}^2 \text{ (compressive)}}$$

2. A masonry dam of base width 3.25m and and top width 1.5m height 8 m retains water up to 7 m.Determine maximum and minimum stresses at base assuming unit length of dam. Unit weight of masonry $\gamma_m = 24$ kN/m^3.Unit weight of water $\gamma_w = 9.81$ kN/m^3

Solution:



Step 1: Geometry of dam section

The dam cross-section is a **trapezium**.

Area of trapezium:

$$A = \frac{(\text{Top width} + \text{Base width})}{2} \times H$$
$$A = \frac{(1.5 + 3.25)}{2} \times 8 = 2.375 \times 8 = 19 \text{ m}^2$$

Step 2: Weight of dam (W)

$$W = A \times \gamma_m$$
$$W = 19 \times 24 = 456 \text{ kN}$$

This weight acts **vertically downward through the centroid** of the trapezium.

Step 3: Water pressure force (P)

Horizontal water pressure on vertical face:

$$P = \frac{1}{2}\gamma_w h^2$$
$$P = \frac{1}{2} \times 9.81 \times 7^2$$
$$P = 240.35 \text{ kN}$$

Point of action of P

Water pressure acts at:

$$\frac{h}{3} = \frac{7}{3} = 2.33 \text{ m above base}$$

Step 4: Moment calculations about center of base

Moment due to water pressure

$$M_P = P \times 2.33$$
$$M_P = 240.35 \times 2.33 = 560.0 \text{ kN\cdotpm}$$

Step 5: Eccentricity (e)

$$e = \frac{M}{W}$$
$$e = \frac{560}{456} = 1.23 \text{ m}$$

Step 6: Check base stress formula

Base width $B = 3.25$ m

Direct stress:

$$\sigma_0 = \frac{W}{B} = \frac{456}{3.25} = 140.3 \text{ kN/m}^2$$

Bending stress:

$$\sigma_b = \frac{6We}{B^2}$$
$$\sigma_b = \frac{6 \times 456 \times 1.23}{(3.25)^2} = 318.7 \text{ kN/m}^2$$

Step 7: Maximum and minimum stresses at base

Maximum stress (toe)

$$\sigma_{\max} = \sigma_0 + \sigma_b$$
$$\sigma_{\max} = 140.3 + 318.7 = \boxed{459.0 \text{ kN/m}^2}$$

Minimum stress (heel)

$$\sigma_{\min} = \sigma_0 - \sigma_b$$

$$\sigma_{\min} = 140.3 - 318.7 = \boxed{-178.4\ \mathrm{kN/m^2}}$$

UNIT-II

Slope and Deflection

A) 2 Marks Questions (with Answers)

1. Define slope of a beam.

Answer: Slope is the angle made by tangent to elastic curve with horizontal at any point.
It is denoted by symbol- θ or dy//dy (no unit)
2. Define deflection.

Answer: Deflection is vertical displacement of a point on beam from original position.
It is denoted by symbol- δ or y (m or mm)
3. Write relation between BM and curvature.

Answer: M/I = E/R,

Were, R-radius of curvature.
M- bending moment
E- Youngs modulus
I- Moment of inertia
4. Write differential equation of elastic curve.

Answer: EI d^2y/dx^2 = M(x).

Where, M(x)- bending moment at xx section.
EI- Flexural rigidity.

5. What is stiffness of beam?

Answer: Stiffness is resistance offered by beam to deflection, proportional to EI.

6. State maximum deflection for simply supported beam with UDL.

Answer: $\delta max = 5wL^4/(384EI)$.

7. State maximum deflection for cantilever with point load at free end.

Answer:$\delta max = WL^3/(3EI)$.

8. What is Macaulay's method?

Answer: It is a method using bracket functions to write bending moment equation for beams with discontinuous loads.

9. Mention boundary conditions for cantilever beam.

Answer: At fixed end: when distance X=0,then slope dy/dx=0 and deflection y=0,
At free end: when distance X=L, then slope dy/dx=maximum and deflection y=maximum
10. Mention boundary conditions for simply supported beam.

Answer: At supports: deflection y=0, Slope dy/dx= Maximum

At Midpoint: deflection y=Maximum, dy/dx= 0

B) 5 Marks Questions (with Solutions)

1. Find slope at free end of cantilever of length L carrying point load W at free end.

Given

Cantilever beam of length **L**

Point load **W** acting at the **free end**

Flexural rigidity = **EI**

Find **slope at free end?**

Solution: -

Step 1: Bending moment equation

Take origin at the **fixed end**.

At a section at distance **x** from fixed end:

$$M_x = -W(L - x)$$

(Negative sign because bending is hogging)

Step 2: Differential equation of elastic curve

$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{d^2y}{dx^2} = -W(L - x)$$

Step 3: First integration (Slope)

$$EI\frac{dy}{dx} = -W(Lx - \frac{x^2}{2}) + C_1$$

Step 4: Apply boundary condition at fixed end

At **fixed end (x = 0)**:

$$\frac{dy}{dx} = 0$$

$$0 = 0 + C_1 \Rightarrow C_1 = 0$$

So,

$$EI\frac{dy}{dx} = -W(Lx - \frac{x^2}{2})$$

Step 5: Slope at free end

At **free end**, $x = L$:

$$EI\theta = -W(L^2 - \frac{L^2}{2})$$

$$EI\theta = -\frac{WL^2}{2}$$

$$\boxed{\theta = \frac{WL^2}{2EI}}$$

(negative sign indicates clockwise rotation; magnitude is written in exams)

$$\boxed{\text{Slope at free end of cantilever } (\theta) = \frac{WL^2}{2EI}}$$

2. Find deflection at free end of cantilever carrying UDL w over entire span.

Given

Cantilever beam of length **L**
Uniformly distributed load **w** (per unit length) over entire span
Flexural rigidity = **EI**
Find **deflection at free end?**

Solution: -

Step 1: Bending moment at a section

Take origin at the **fixed end**.
At a section at distance **x** from fixed end:

$$M_x = -\frac{w}{2}(L - x)^2$$

Step 2: Differential equation of elastic curve

$$EI\frac{d^2y}{dx^2} = M_x$$
$$EI\frac{d^2y}{dx^2} = -\frac{w}{2}(L - x)^2$$

Step 3: First integration (Slope)

$$EI\frac{dy}{dx} = -\frac{w}{2}(L^2x - Lx^2 + \frac{x^3}{3}) + C_1$$

Step 4: Second integration (Deflection)

$$EIy = -\frac{w}{2}(\frac{L^2x^2}{2} - \frac{Lx^3}{3} + \frac{x^4}{12}) + C_1x + C_2$$

Step 5: Apply boundary conditions at fixed end

At **fixed end (x = 0)**:

- $y = 0$
- $\frac{dy}{dx} = 0$

From slope equation:

$$C_1 = 0$$

From deflection equation:

$$C_2 = 0$$

Step 6: Deflection at free end

At **free end**, $x = L$:

$$EIy = -\frac{w}{2}(\frac{L^4}{2} - \frac{L^4}{3} + \frac{L^4}{12})$$

Simplifying:

$$EIy = -\frac{wL^4}{8}$$

$$\boxed{y = \frac{wL^4}{8EI}}$$

(Negative sign indicates downward deflection; magnitude is written)

$$\boxed{\text{Deflection at free end} = \frac{wL^4}{8EI}}$$

3. Explain double integration method.

Solution: -

Double Integration Method-

The **double integration method** is an analytical method used to determine the **slope and deflection of beams** subjected to bending.

Principle-

From bending theory,

$$\frac{M}{I} = \frac{E}{R}$$

For small deflections,

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Hence,

$$EI\frac{d^2y}{dx^2} = M(x)$$

Procedure-

1. **Write bending moment equation**
   Determine the bending moment $M(x)$at a section $x$from a reference end.

2. **Form differential equation of elastic curve**

$$EI\frac{d^2y}{dx^2} = M(x)$$

3. **Integrate twice**

   - First integration gives **slope**:

$$EI\frac{dy}{dx} = \int M(x)\,dx + C_1$$

   - Second integration gives **deflection**:

$$EIy = \int \left(\frac{dy}{dx}\right)dx + C_2$$

4. **Apply boundary conditions**
   Constants $C_1$and $C_2$are found using known conditions:
   - At fixed end: $y = 0,\ \frac{dy}{dx} = 0$
   - At simply supported end: $y = 0$
   - At point of maximum deflection: $\frac{dy}{dx} = 0$
5. **Determine slope and deflection**
   Substitute the required value of $x$to obtain slope or deflection at the desired point.

4. Explain Macaulay's method with example.

Solution: -
Macaulay's Method-
Macaulay's method is an extension of the double integration method used to determine **slope and deflection of beams carrying discontinuous or point loads**.
Principle-
In this method, bending moment is expressed using **Macaulay's brackets**:

$$\langle x - a\rangle = \{\begin{matrix} 0, & x < a \\ x - a, & x \geq a \end{matrix}$$

This allows the entire beam to be represented by **a single bending moment equation**.
Procedure-
1. **Find support reactions** using equilibrium equations.
2. **Write bending moment equation** for the whole beam using Macaulay brackets.
3. **Form differential equation**:
$$EI\frac{d^2y}{dx^2} = M(x)$$
4. **Integrate twice** to obtain slope and deflection.
5. **Apply boundary conditions** to find constants of integration.

Example-

Simply supported beam of span Lcarrying a point load Wat mid-span.

Bending Moment Equation
Let $x$be distance from left support.

$$M_x = \frac{W}{2}x - W\langle x - \frac{L}{2}\rangle$$

Differential Equation

$$EI\frac{d^2y}{dx^2} = \frac{W}{2}x - W\langle x - \frac{L}{2}\rangle$$

Integration
First integration (Slope):

$$EI\frac{dy}{dx} = \frac{W}{4}x^2 - \frac{W}{2}\langle x - \frac{L}{2}\rangle^2 + C_1$$

Second integration (Deflection):

$$EIy = \frac{W}{12}x^3 - \frac{W}{6}\langle x - \frac{L}{2}\rangle^3 + C_1x + C_2$$

Boundary Conditions

At supports $x = 0$and $x = L$:

$$y = 0$$

Solving,

$$C_1 = -\frac{WL^2}{16}, C_2 = 0$$

Maximum Deflection (at mid-span)

$$\boxed{y_{max} = \frac{WL^3}{48EI}}$$

5. A simply supported beam carries central point load W. Write expression for max deflection.

Given-

Simply supported beam of span **L**
Central point load **W**
Flexural rigidity = **EI**
Find **maximum deflection?**

Solution: -

Step 1: Reactions at supports

By symmetry,

$$R_A = R_B = \frac{W}{2}$$

Step 2: Bending moment equation

Consider **left half** of the beam.

Take origin at **left support A**.

For a section at distance **x** from A ($0 \leq x \leq L/2$):

$$M_x = \frac{W}{2}x$$

Step 3: Differential equation of elastic curve

$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{d^2y}{dx^2} = \frac{W}{2}x$$

Step 4: First integration (Slope)

$$EI\frac{dy}{dx} = \frac{W}{4}x^2 + C_1$$

Step 5: Second integration (Deflection)

$$EIy = \frac{W}{12}x^3 + C_1x + C_2$$

Step 6: Apply boundary conditions

(i) At support A, $x = 0$:

$$y = 0 \Rightarrow C_2 = 0$$

(ii) At centre, slope is zero (symmetry):

$$\frac{dy}{dx} = 0 \text{ at } x = \frac{L}{2}$$

$$0 = \frac{W}{4}(\frac{L}{2})^2 + C_1$$

$$C_1 = -\frac{WL^2}{16}$$

Step 7: Maximum deflection at centre

At $x = \frac{L}{2}$:

$$EIy_{max} = \frac{W}{12}(\frac{L}{2})^3 - \frac{WL^2}{16}(\frac{L}{2})$$

$$EIy_{max} = -\frac{WL^3}{48}$$

$$\boxed{\text{Maximum deflection} = \frac{WL^3}{48EI}}$$

C) 10 Marks Questions (with Solutions)

1. A simply supported beam of span 6 m carries UDL 10 kN/m. Find maximum deflection and slope at supports. Take E=2×10⁵ N/mm², I=8×10⁸ mm⁴.

Given: $E = 2 \times 10^5 \text{ N/mm}^2, I = 8 \times 10^8 \text{ mm}^4, Load = 10\frac{kN}{m}$, Span = 6m

Solution:

STEP 1: Convert units

Span:

$$L = 6\,\text{m} = 6000\,\text{mm}$$

UDL:

$$w = 10\,\text{kN/m} = 10\,\text{N/mm}$$

STEP 2: Support reactions

Total load:

$$wL = 10 \times 6000 = 60000\,\text{N}$$

Since loading is symmetrical:

$$R_A = R_B = \frac{60000}{2} = 30000\,\text{N}$$

STEP 3: Bending moment equation

Take section at distance x from left support A.

$$M(x) = R_A x - \frac{wx^2}{2}$$

Substitute values:

$$M(x) = 30000x - \frac{10x^2}{2}$$

$$\boxed{M(x) = 30000x - 5x^2}$$

STEP 4: Differential equation of bending

$$EI\frac{d^2y}{dx^2} = M(x)$$

$$EI\frac{d^2y}{dx^2} = 30000x - 5x^2$$

STEP 5: First integration (Slope)

$$EI\frac{dy}{dx} = \int (30000x - 5x^2)\,dx$$

$$EI\frac{dy}{dx} = 15000x^2 - \frac{5x^3}{3} + C_1$$

STEP 6: Second integration (Deflection)

$$EIy = \int (15000x^2 - \frac{5x^3}{3} + C_1)dx$$

$$EIy = 5000x^3 - \frac{5x^4}{12} + C_1x + C_2$$

STEP 7: Apply boundary conditions

Boundary condition 1

At left support A: $x = 0$, deflection$=y = 0$

$$0 = 0 + 0 + 0 + C_2 \Rightarrow \boxed{C_2 = 0}$$

Boundary condition 2

When,$x = 6000/2$, slope $=$ d$y/dx = 0$

$$C_1 = -15000(3000)^2 + \frac{5(3000)^3}{3}$$

$$\boxed{C_1 = -9 \times 10^{10}}$$

STEP 8: Final slope equation

$$EI\frac{dy}{dx} = 15000x^2 - \frac{5x^3}{3} - 9 \times 10^{10}$$

STEP 9: Slope at supports

First calculate:

$$EI = (2 \times 10^5)(8 \times 10^8) = 1.6 \times 10^{14}$$

- **Slope at left support $A(x = 0)$**

$$\theta_A = \frac{dy}{dx} = \frac{-9 \times 10^{10}}{1.6 \times 10^{14}}$$
$$\boxed{\theta_A = -5.625 \times 10^{-4} \text{ radians}}$$

- **Slope at right support B**

By symmetry:

$$\boxed{\theta_B = +5.625 \times 10^{-4} \text{ radians}}$$

STEP 10: Maximum deflection

For simply supported beam with UDL, max deflection occurs at **mid-span**:

$$x = \frac{L}{2} = 3000 \text{ mm}$$

Substitute into deflection equation

$$EIy = 5000(3000)^3 - \frac{5(3000)^4}{12} - 9 \times 10^{10}(3000)$$

Calculate terms carefully

$$5000(3000)^3 = 1.35 \times 10^{14}$$
$$\frac{5(3000)^4}{12} = 3.375 \times 10^{13}$$
$$9 \times 10^{10} \times 3000 = 2.7 \times 10^{14}$$

Combine

$$EIy = 1.35 \times 10^{14} - 3.375 \times 10^{13} - 2.7 \times 10^{14}$$
$$EIy = -1.6875 \times 10^{14}$$

Divide by EI

$$y = \frac{-1.6875 \times 10^{14}}{1.6 \times 10^{14}}$$
$$\boxed{y_{max} = -1.055 \text{ mm}}$$

Final Answer-

- **Slope at supports**

$$\boxed{\theta_A = \theta_B = 5.625 \times 10^{-4} \text{ radians}}$$

- **Maximum deflection at mid-span**

$$\boxed{y_{max} = 1.055 \text{ mm downward}}$$

---

2. A cantilever beam 3 m long carries point load 20 kN at free end. Find slope and deflection at free end. Take E=2×10^5 N/mm^2, I=6×10^8 mm^4.

Given data

Length of cantilever beam, $L = 3\text{ m} = 3000\text{ mm}$
Point load at free end, $W = 20\text{ kN} = 20{,}000\text{ N}$
Young's modulus, $E = 2 \times 10^5\text{ N/mm}^2$
Moment of inertia, $I = 6 \times 10^8\text{ mm}^4$

Solution:

Step 1: Bending moment equation

Take **x** as the distance from the **fixed end**.

For a cantilever with a point load at the free end,

$$M(x) = -W(L - x)$$
$$M(x) = -20{,}000(3000 - x)$$

Step 2: Differential equation of bending

$$EI\frac{d^2y}{dx^2} = M(x)$$
$$EI\frac{d^2y}{dx^2} = -20{,}000(3000 - x)$$

Step 3: First integration (slope equation)

$$EI\frac{dy}{dx} = \int -20{,}000(3000 - x)\,dx$$
$$EI\frac{dy}{dx} = -20{,}000(3000x - \frac{x^2}{2}) + C_1$$

Boundary condition at fixed end

At $x = 0$, slope = 0

$$0 = -20{,}000(0) + C_1 \Rightarrow C_1 = 0$$

So,

$$EI\frac{dy}{dx} = -20{,}000(3000x - \frac{x^2}{2})$$

Step 4: Second integration (deflection equation)

$$EIy = \int -20{,}000(3000x - \frac{x^2}{2})dx$$
$$EIy = -20{,}000(1500x^2 - \frac{x^3}{6}) + C_2$$

Boundary condition at fixed end

At $x = 0$, deflection = 0

$$C_2 = 0$$

So,

$$EIy = -20{,}000(1500x^2 - \frac{x^3}{6})$$

Step 5: Slope at free end

At free end, $x = L = 3000$ mm

$$\theta = \frac{dy}{dx} = \frac{-20{,}000(3000x - \frac{x^2}{2})}{EI}$$

Substitute values:

$$\theta = \frac{-20{,}000(3000 \times 3000 - \frac{3000^2}{2})}{(2 \times 10^5)(6 \times 10^8)}$$
$$\theta = \frac{-20{,}000(9{,}000{,}000 - 4{,}500{,}000)}{1.2 \times 10^{14}}$$
$$\theta = \frac{-9 \times 10^{10}}{1.2 \times 10^{14}}$$
$$\boxed{\theta = -7.5 \times 10^{-4} \text{ radians}}$$

(Negative sign indicates downward rotation)

Step 6: Deflection at free end

$$y = \frac{-20{,}000(1500x^2 - \frac{x^3}{6})}{EI}$$

At $x = 3000$ mm:

$$y = \frac{-20{,}000(1500 \times 3000^2 - \frac{3000^3}{6})}{1.2 \times 10^{14}}$$
$$y = \frac{-20{,}000(13.5 \times 10^9 - 4.5 \times 10^9)}{1.2 \times 10^{14}}$$
$$y = \frac{-1.8 \times 10^{14}}{1.2 \times 10^{14}}$$
$$\boxed{y = -1.5 \text{ mm}}$$

<u>Final Answers</u>

- **Slope at free end**

$$\boxed{\theta = 7.5 \times 10^{-4} \text{ radians (downward)}}$$

Deflection at free end

$$\boxed{y = 1.5 \text{ mm (downward)}}$$

UNIT-III

Determinate and Indeterminate Structures (Fixed and Continuous Beam)

A) 2 Marks Questions (with Answers)

1. Define statically determinate structure.

Answer: A structure where reactions and member forces can be found using equilibrium equations only.

2. Define statically indeterminate structure.

Answer: A structure having more unknowns than equilibrium equations, requiring compatibility conditions.

3. What is fixed beam?

Answer: A beam fixed at both ends such that rotation and deflection at ends are zero.

4. State one advantage of fixed beam.

Answer: Fixed beam has smaller maximum bending moment and deflection than simply supported beam.

5. State one disadvantage of fixed beam.

Answer: It develops end moments and is sensitive to support settlement.

6. Define fixed end moment (FEM).

Answer: Moment developed at fixed supports due to external loading.

7. Write FEM for point load at midspan.

Answer: FEM = WL/8 at both ends (hogging).

8. Write FEM for UDL over entire span.

Answer: FEM = $wL^2/12$ at both ends (hogging).

9. State Clapeyron's theorem.

Answer: Sum of moments in continuous beam spans related by 3-moment equation considering loads and spans.

10. Define influence line diagram. Answer: ILD shows variation of response (reaction, shear, moment) at a point due to moving unit load.B5 Marks Questions (with Solutions)

Q1. Explain the concept of statically determinate and statically indeterminate structures.

Answer:
A structure is said to be statically determinate when all the support reactions and internal forces can be determined completely by using only the equations of static equilibrium, namely $\Sigma Fx = 0$, $\Sigma Fy = 0$ and $\Sigma M = 0$. In such structures, the number of unknown reactions is exactly equal to the number of available equilibrium equations. These structures are simple to analyse and do not develop additional stresses due to temperature variation or support settlement. Examples of statically determinate structures include simply supported beams, cantilever beams and three-hinged arches.

A structure is said to be statically indeterminate when the number of unknown reactions exceeds the number of equilibrium equations. In this case, equilibrium equations alone are insufficient, and additional equations based on compatibility of deformation and material properties are required. Fixed beams, continuous beams and propped cantilevers are common examples. Although analysis of indeterminate structures is more complex, they are generally stronger, stiffer and more economical than determinate structures.

Q2. Explain the concept of fixity and discuss the advantages and disadvantages of a fixed beam over a simply supported beam.

Answer:
Fixity refers to the condition in which the supports of a beam restrain both rotation and translation. In a fixed beam, the ends are rigidly embedded in the supports, preventing angular rotation. Due to this restraint, bending moments develop at the supports, known as negative or hogging moments.

The effect of fixity is a reduction in the maximum bending moment and deflection at mid-span, thereby increasing the stiffness of the beam. Fixed beams are advantageous compared to simply supported beams because they experience smaller maximum bending moments, lesser deflection and better load-carrying capacity for the same span and loading. This often results in a more economical beam section.

However, fixed beams also have disadvantages. They are statically indeterminate and sensitive to temperature changes, support settlement and shrinkage. Construction of perfectly fixed supports is difficult in practice, and any unexpected movement at supports can induce additional stresses.

Q3. State and explain the principle of superposition. Mention its limitations.

Answer:
The principle of superposition states that the total effect (such as bending moment, shear force or deflection) produced by several loads acting simultaneously on a structure is equal to the algebraic sum of the effects produced by each load acting individually.

This principle is widely used in structural analysis, particularly for beams subjected to multiple types of loading. For example, the bending moment at a section due to a point load and a uniformly distributed load can be obtained by calculating the bending moment due to each load separately and then adding them algebraically.

The principle of superposition is valid only when the structure behaves linearly, the material obeys Hooke's law, and deformations are small. It is not applicable in cases involving large deformations, plastic behaviour or non-linear materials.

Q4. Explain fixed end moments and obtain expressions for a fixed beam subjected to (a) a point load at mid-span and (b) UDL over the entire span.

Answer:
Fixed End Moments (FEM) are the moments developed at the ends of a beam when both ends are fixed against rotation. In a fixed beam, the slopes at both ends are zero, and this condition is used to obtain fixed end moments.

(a) For a fixed beam of span L subjected to a point load W at the mid-span, equal hogging moments develop at both ends. Using the condition of zero slope at the supports, the fixed end moments are:

FEM at left end = FEM at right end = $-WL/8$

(b) For a fixed beam carrying a uniformly distributed load w per unit length over the entire span, the fixed end moments at both supports are:

FEM at left end = FEM at right end = $-wL^2/12$

These moments play an important role in analysing fixed and continuous beams using standard formulae.

Q5. Define a continuous beam. Explain the effect of continuity and the nature of moments induced due to continuity.

Answer:
A continuous beam is a beam that rests on more than two supports. Due to continuity, the beam is restrained at intermediate supports, resulting in redistribution of bending moments along the span.

The effect of continuity is a reduction in maximum bending moment and deflection compared to a simply supported beam. Negative or hogging moments develop over the intermediate supports, while positive or sagging moments occur within the spans. Because of these negative moments, the beam curves upward near the supports and downward at mid-span.

Continuity improves structural efficiency and load distribution. Continuous beams are commonly used in bridges, multi-span floor systems and long-span girders, where economy and stiffness are important considerations.

Q6. State Clapeyron's theorem of three moments and mention its applications and limitations.

Answer:
Clapeyron's theorem of three moments relates the bending moments at three consecutive supports of a continuous beam. It provides a relationship between the support moments by considering the geometry and loading of the beam.

The theorem is applicable for beams with supports at the same level and constant or known moment of inertia. It is used mainly for analysing continuous beams with a maximum of three spans and two unknown support moments.

The theorem is applicable for concentrated loads and uniformly distributed loads over the entire span. However, it is not suitable for beams with varying support levels or rapidly changing moment of inertia unless modifications are made.

C) 10 Marks Questions (with Solutions)
Problem: 2

A fixed beam AB of span 6 m carries a uniformly distributed load of 20 kN/m over the entire span. Determine:

1. **Fixed end moments**
2. **Reactions at the supports**
3. **Maximum bending moment**

Solution:
Given:

- Span, $L = 6$ m
- UDL, $w = 20$ kN/m

Step 1: Fixed End Moments

For a fixed beam carrying UDL over the entire span:

FEM at A = FEM at B = $-wL^2 / 12$

$= -(20 \times 6^2) / 12$
$= -(20 \times 36) / 12$
$= -60$ kN·m

Step 2: Reactions at Supports

Total load on beam = wL = 20 × 6 = 120 kN

Due to symmetry: Reaction at A = Reaction at B = 120 / 2 = 60 kN

Step 3: Maximum Bending Moment

For a fixed beam with UDL, maximum positive bending moment occurs at mid-span:

$M_{max} = wL^2 / 24$
= (20 × 36) / 24
= 30 kN·m

Answer:

- Fixed end moments = −60 kN·m
- Reactions at supports = 60 kN each
- Maximum sagging moment = 30 kN·m

Problem:2
A fixed beam of span 4 m carries a point load of 80 kN at its mid-span. Determine the fixed end moments, support reactions and sketch the bending moment diagram.

Solution:
Given:

- Span, L = 4 m
- Point load, W = 80 kN

Step 1: Fixed End Moments

For a point load at mid-span:

FEM at A = FEM at B = −WL / 8

= −(80 × 4) / 8
= −40 kN·m

Step 2: Support Reactions

Total load = 80 kN

By symmetry: Reaction at A = Reaction at B = 80 / 2 = 40 kN

Step 3: Bending Moment at Mid-Span

Positive bending moment at mid-span:

Mmid = WL / 8
= (80 × 4) / 8
= 40 kN·m

Answer:

- Fixed end moments = −40 kN·m
- Reactions = 40 kN each
- Maximum sagging moment at mid-span = 40 kN·m

Problem:3
A continuous beam ABC has two equal spans AB = BC = 5 m. The beam carries a uniformly distributed load of 10 kN/m over both spans. Supports are at the same level and EI is constant. Determine the bending moment at the intermediate support B.

Solution:
Given:

- L1 = L2 = 5 m
- w = 10 kN/m
- MA = MC = 0 (simply supported ends)

Step 1: Three Moment Equation

For spans AB and BC:

MA·L1 + 2MB(L1 + L2) + MC·L2
= −6 [(Area of BM of AB / L1) + (Area of BM of BC / L2)]

Since MA = MC = 0:

$2MB(5 + 5) = -6 [(wL^4 / 12) / L + (wL^4 / 12) / L]$

Step 2: Substitute Values

$20MB = -6 [(10 \times 5^3 / 12) + (10 \times 5^3 / 12)]$

20MB = −6 × (2 × 104.17)

20MB = −1250

MB = −62.5 kN·m

Answer:
Bending moment at intermediate support B = −62.5 kN·m (hogging)

UNIT-IV

Moment Distribution Method

A) 2 Marks Questions (with Answers)

1. What is moment distribution method?

Answer: A method to analyze indeterminate beams by distributing unbalanced moments at joints based on stiffness.

2. Define carry over factor.

Answer: Carry over factor is ratio of moment carried over to far end when near end is fixed (usually 1/2).

3. Define stiffness factor.

Answer: Stiffness = moment required to produce unit rotation at joint with other end condition fixed/free.

4. Write stiffness for member with far end fixed.

Answer: K = 4EI/L.

5. Write stiffness for member with far end hinged.

Answer: K = 3EI/L.

6. Define distribution factor.

Answer: DF = stiffness of member / sum of stiffness at joint.

7. What is sign convention in MDM?

Answer: Clockwise moment at joint is taken as positive (commonly used).

8. What is balancing moment?

Answer: Moment applied at joint to bring algebraic sum of moments to zero.

9. State one advantage of MDM.

Answer: It is simple and suitable for continuous beams with several spans.

10. What is portal frame?

Answer: A rigid frame consisting of columns and beam forming bays and stories.

B) 5 Marks Questions (with Solutions)

1. Explain the Moment Distribution Method. State its basic assumptions and advantages.

Answer:
The **Moment Distribution Method** is an iterative method of structural analysis developed by **Hardy Cross** for analysing statically indeterminate structures such as continuous beams and rigid frames. In this method, the structure is first assumed to be fully fixed at all joints. The fixed end moments due to applied loads are calculated, and then the moments at joints are successively released and redistributed until equilibrium is achieved.

The method is based on the concept that when a joint rotates, the unbalanced moment at that joint is distributed among the connected members in proportion to their stiffness. Half of the distributed moment is carried over to the far end of the member, provided the far end is fixed.

Assumptions:

- Joints are rigid and capable of transmitting moments
- Members behave elastically and obey Hooke's law
- Deformations are small
- Supports are at the same level

Advantages:

- Simple and systematic procedure
- Suitable for hand calculations
- Particularly useful for continuous beams with few spans

2. Explain the sign convention used in the Moment Distribution Method.

Answer:
In the Moment Distribution Method, a consistent sign convention is essential to avoid errors during calculation and redistribution of moments. Generally, **clockwise moments are taken as positive**, while **anticlockwise moments are taken as negative**. Hogging moments (which cause compression at the top fibers) are usually considered negative, whereas sagging moments are considered positive.

At a joint, if the algebraic sum of moments is not zero, the joint is said to be unbalanced. This unbalanced moment is then distributed among the connected members according to their distribution factors. Throughout the moment distribution table, the same sign convention must be maintained for fixed end moments, distributed moments and carried-over moments.

A clear and consistent sign convention helps in correctly determining final end moments and ensures accurate bending moment diagrams.

3. Define stiffness factor, distribution factor and carry-over factor used in the Moment Distribution Method.

Answer:
The **stiffness factor** of a member is defined as the moment required to produce unit rotation at one end of the member while the other end is restrained. It indicates the relative rigidity of the member. For a prismatic member of length L and flexural rigidity EI:

- Far end fixed: K = 4EI/L
- Far end hinged: K = 3EI/L

The **distribution factor** is the proportion of the unbalanced moment at a joint that is distributed to a particular member. It is given by the ratio of the stiffness of that member to the sum of stiffnesses of all members meeting at the joint. The sum of distribution factors at any joint is always equal to unity.

The **carry-over factor** is the ratio of the moment induced at the far end of a member to the moment applied at the near end. For a prismatic member with the far end fixed, the carry-over factor is 1/2. If the far end is hinged, the carry-over factor is zero.

4. Explain the step-by-step procedure of the Moment Distribution Method for analysing continuous beams.

Answer:
The procedure of the Moment Distribution Method involves the following steps:

1. Assume all joints of the beam to be fixed and calculate the fixed end moments due to applied loads.
2. Calculate the stiffness factor for each member considering the end conditions.
3. Determine the distribution factor for each member at every joint.
4. At each joint, find the unbalanced moment by algebraic summation of moments.
5. Distribute the unbalanced moment among the connected members using distribution factors.
6. Carry over half of the distributed moments to the far ends of the members.
7. Repeat the distribution and carry-over process until the unbalanced moments become negligibly small.
8. Add all moments algebraically to obtain the final end moments.

This method gives accurate results for continuous beams with limited spans and is widely used in structural analysis.

5. Explain the application of the Moment Distribution Method to continuous beams with same or different moment of inertia.

Answer:
The Moment Distribution Method can be applied effectively to continuous beams having either the same or different moments of inertia. When all members have the same moment of inertia, the stiffness factors depend only on the span length, which simplifies the calculation of distribution factors.

When the moment of inertia differs from span to span, the stiffness factor for each member is calculated using its respective EI value. Members with higher stiffness attract a larger share of the unbalanced moment at the joint. The remaining steps of the method remain unchanged.

This flexibility makes the Moment Distribution Method suitable for analysing beams with varying cross-sections, provided the number of spans is limited and supports are at the same level.

6. Define a portal frame. Explain symmetrical portal frames with neat explanation.

Answer:
A **portal frame** is a rigid frame structure consisting of vertical columns and horizontal beams connected by moment-resisting joints. Portal frames are widely used in industrial buildings, warehouses and bridge structures due to their ability to resist both vertical and lateral loads.

A **symmetrical portal frame** has equal column heights, equal bay widths and symmetrical loading. Due to symmetry in geometry and loading, bending moments, shear forces and reactions on both sides of the frame are equal. This significantly simplifies the analysis.

Symmetrical portal frames exhibit uniform deflected shapes and are commonly analysed using approximate or exact methods such as moment distribution or slope-deflection methods.

C) 10 Marks Questions (with Solutions)

1: Continuous Beam Analysis by Moment Distribution Method (10 Marks)

Problem:1
A continuous beam ABC has two spans AB = 6 m and BC = 6 m. The beam carries a uniformly distributed load of 20 kN/m over both spans. The supports A and C are fixed and B is an intermediate rigid joint. The flexural rigidity EI is constant for all members. Determine the final end moments at supports A, B and C using the **Moment Distribution Method**.

Solution:

Step 1: Fixed End Moments (FEM)

For a fixed beam carrying UDL over entire span:

FEM = $-wL^2 / 12$

For span AB:

- FEM at A = $-(20 \times 6^2) / 12 = -60$ kN·m
- FEM at B (from AB) = +60 kN·m

For span BC:

- FEM at B (from BC) = −60 kN·m
- FEM at C = +60 kN·m

Step 2: Stiffness Factors

For all members (far end fixed):

K = 4EI / L = 4EI / 6

At joint B:

- Stiffness of BA = 4EI / 6
- Stiffness of BC = 4EI / 6

Step 3: Distribution Factors at Joint B

Total stiffness at B = (4EI/6 + 4EI/6) = 8EI/6

Distribution factor for BA = (4EI/6) / (8EI/6) = 0.5
Distribution factor for BC = 0.5

Step 4: Moment Distribution at Joint B

Initial moments at B:

- From AB = +60 kN·m
- From BC = −60 kN·m

Unbalanced moment at B:

$= +60 - 60 = 0$

Hence, no moment distribution is required at joint B.

Step 5: Final End Moments

- Moment at A = −60 kN·m
- Moment at B = 0 kN·m
- Moment at C = −60 kN·m

Answer:
Final end moments are:

- $M_A = -60$ kN·m
- $M_B = 0$ kN·m
- $M_C = -60$ kN·m

Numerical 2: Symmetrical Portal Frame Analysis (10 Marks)

Problem:2
A symmetrical portal frame has two columns each 4 m high and a beam of span 6 m. All joints are rigid. The frame carries a uniformly distributed load of 15 kN/m on the beam. Flexural rigidity EI is same for all members. Determine the bending moment at the beam–column joints using the **Moment Distribution Method**.

Solution:

Step 1: Fixed End Moments

For beam carrying UDL:

$FEM = -wL^2 / 12 = -(15 \times 6^2) / 12 = -45$ kN·m

Thus:

- Beam left end = −45 kN·m
- Beam right end = −45 kN·m

Columns have no external load:

- FEM at column ends = 0

Step 2: Stiffness Factors

Beam stiffness (both ends fixed): K = 4EI / 6

Column stiffness (base fixed, top joint rotating): K = 4EI / 4

Step 3: Distribution Factors at Left Joint

Total stiffness = (4EI/6 + 4EI/4)

Distribution factor of beam = (4EI/6) / (4EI/6 + 4EI/4) = 0.4
Distribution factor of column = 0.6

(Same distribution factors apply at right joint due to symmetry)

Step 4: Moment Distribution

Unbalanced moment at left joint = −45 kN·m

Distributed moments:

- To beam = −45 × 0.4 = −18 kN·m
- To column = −45 × 0.6 = −27 kN·m

Carry-over moments:

- Beam to right joint = −9 kN·m
- Column to base = −13.5 kN·m

Similar distribution occurs at the right joint.

Step 5: Final End Moments

Final bending moments at beam–column joints:

≈ −63 kN·m (hogging)

Answer:
The bending moment at each beam–column joint of the symmetrical portal frame is approximately **−63 kN·m**.

UNIT-V

Simple Trusses

A) 2 Marks Questions (with Answers)

1. Define truss.

Answer: A framework of members connected at joints, carrying loads at joints only.

2. What is simple truss?

Answer: A truss that can be built by adding one joint and two members at a time, satisfying $m=2j-3$.

3. Name any four types of trusses.

Answer: Pratt, Howe, Fink, King post.

4. What is method of joints?

Answer: A method to find member forces by applying equilibrium at each joint.

5. What is method of sections?

Answer: A method to find member forces by cutting truss and applying equilibrium to part.

6. State assumption used in truss analysis.

Answer: Members are pin-jointed and loads act only at joints.

7. How to find reactions of simply supported truss?

Answer: Using $\Sigma M=0$, $\Sigma V=0$, $\Sigma H=0$ for entire truss.

8. What is zero force member?

Answer: Member carrying zero force due to geometry/loading conditions.

9. Tension and compression in truss members are found by?

Answer: By assuming member forces away from joint (tension) and using equilibrium.

10. Write condition for determinacy of plane truss.

Answer: m = 2j – 3.

B) 5 Marks Questions (with Solutions)

1. Explain method of joints with steps.

Solution:

The method of joints analyzes a truss by isolating one joint at a time and applying the equations of static equilibrium to that joint.

Step-by-Step Procedure

Step 1: Check if the truss is statically determinate

Before solving, make sure the truss can be solved using statics alone:

$$m + r = 2j$$

Where:

- m= number of members
- r= number of reactions
- j= number of joints

If this condition is satisfied → method of joints will work.

Step 2: Calculate support reactions

Treat the entire truss as one rigid body and apply equilibrium equations:

$$\sum F_x = 0, \sum F_y = 0, \sum M = 0$$

This gives you the reaction forces at the supports.

Step 3: Choose a joint with only two unknowns

Step 4: Draw the Free Body Diagram (FBD) of the joint

Step 5: Apply equilibrium equations at the joint

Since the joint is in equilibrium:

$$\sum F_x = 0 \sum F_y = 0$$

Solve these two equations to find the unknown member forces.

Step 6: Identify tension or compression

- Positive value → member is in tension
- Negative value → member is in compression

2. Explain method of sections with steps.

Solution:

The method of sections is used to find the force in specific truss members directly, without analyzing the whole truss joint by joint.

1. Determine support reactions
2. Cut the truss through required members
3. Select the simpler section
4. Draw FBD of that section
5. Apply $\sum M = 0$to find one member force
6. Apply $\sum F_x = 0, \sum F_y = 0$for remaining forces
7. Conclude tension or compressio

3.What is king post and queen post trusses. Explain it with example.

Solution:
A king post truss is a simple roof truss in which a single vertical member (king post) is placed at the center, connecting the ridge to the tie beam.

Main features:
- One vertical post at the center
- Suitable for short spans (about 5–8 m)
- Simple and economical design

Example:
- Used in small residential buildings, sheds, and garages

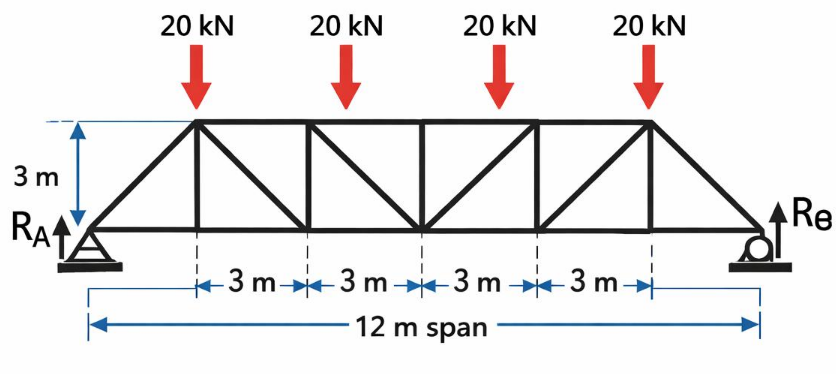
A queen post truss is a roof truss that has two vertical members (queen posts) placed symmetrically on either side of the center.

Main features:
- Two vertical posts
- Can span longer distances than a king post truss (about 8–12 m)
- More stable for wider roofs

Example:

- Used in halls, schools, and larger residential buildings

4.What is Pratt Truss. Give two applications of it. A Pratt truss span 12 m has 4 panels each 3 m, height 3 m. Joint loads 20 kN at each top joint. Calculate support reaction.



Load Diagram

Solution.

A Pratt truss is a type of truss in which the diagonal members slope downward toward the center of the span, while the vertical members are perpendicular to the bottom chord. In this truss, diagonal members are mainly in tension and vertical members are in compression.

Applications (any two):

1. Railway bridges
2. Highway bridges

Solution:
Total load = 20x4 =80 kN.
Reactions R_A=R_B=40kN.

5. Explain Pratt and Howe truss difference.

Solution :

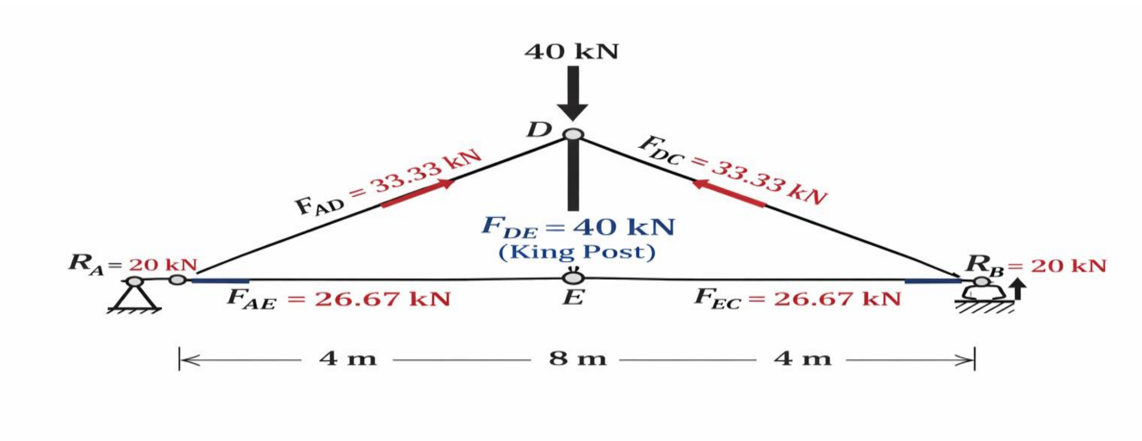
Comparison between Pratt Truss and Howe Truss

Aspect	Pratt Truss	Howe Truss
Direction of diagonals	Slopes **downwards** to center	Slopes **upwards** to center
Diagonal members	Tension	Compression
Vertical members	Compression	Tension
Suitable material	Steel	Timber
Structural efficiency	Better for long spans	Better for short to medium spans
Common use	Steel bridges, railway bridges	Wooden bridges, roofs

C) 10 Marks Questions (with Solutions)

A simply supported truss of span 8 m has a central load 40 kN at top joint. Find reactions and forces in members using method of joints (assume symmetrical king post truss).

Solution:



1. Given data

- Type of truss: **Simply supported, symmetrical king post truss**
- Span = **8 m**
- Central load = **40 kN** (acting downward at the **top joint**)
- Supports:
    - Left support = **Pin (A)**

- Right support = **Roller (B)**

2. Geometry of the truss (Assumption)

Since height is not given, we assume a **standard king post truss**:

- Bottom chord length = 8 m
- Central joint divides span equally → 4 m on each side
- Height of truss = **3 m** (commonly assumed in exams)

So for inclined members:

$$\tan\ \theta = \frac{3}{4}$$
$$\sin\ \theta = \frac{3}{5}, \cos\ \theta = \frac{4}{5}$$

Truss joints and members

- Bottom joints: **A (left), C (right)**
- Top joint: **D**
- Bottom middle joint: **E**

Members:

- AD and DC → inclined members
- AE and EC → bottom chord
- DE → king post (vertical member)

3. Support reactions

Because the truss and loading are **symmetrical**:

$$R_A = R_B$$

Taking vertical equilibrium:

$$R_A + R_B = 40$$
$$R_A = R_B = \boxed{20\text{ kN (upward)}}$$

4. Method of Joints

We start from **Joint D** (top joint) because only **two unknowns** are present.

Joint D

Forces acting at joint D:

- Load = 40 kN ↓
- Force in member **AD** = F_{AD}
- Force in member **DC** = F_{DC}

By symmetry:

$$F_{AD} = F_{DC}$$

Vertical equilibrium at joint D:

$$2F_{AD}\sin\,\theta = 40$$
$$2F_{AD} \times \frac{3}{5} = 40$$
$$F_{AD} = \frac{40 \times 5}{6} = \boxed{33.33\text{ kN}}$$

Nature:
Since members push upward on the joint → **Compression**

$$F_{AD} = F_{DC} = \boxed{33.33\text{ kN (Compression)}}$$

Joint E (middle bottom joint)

Forces at joint E:

- Force in **AE**
- Force in **EC**
- Force in **DE** (vertical)

By symmetry:

$$F_{AE} = F_{EC}$$

Vertical equilibrium at joint E:

Only vertical force is from king post **DE**

$$F_{DE} = 40\text{ kN}$$

Since it pulls the joint upward → **Tension**

$$\boxed{F_{DE} = 40\text{ kN (Tension)}}$$

Joint A

Forces at joint A:

- Reaction = 20 kN ↑
- Member **AD** (known)
- Member **AE**

Vertical equilibrium:

$$20 = F_{AD} \sin\ \theta$$
$$20 = 33.33 \times \frac{3}{5}$$

Horizontal equilibrium:

$$F_{AE} = F_{AD} \cos\ \theta$$
$$F_{AE} = 33.33 \times \frac{4}{5}$$
$$F_{AE} = \boxed{26.67\ \text{kN}}$$

Nature:
Pulling away from joint → **Tension**

$$F_{AE} = F_{EC} = \boxed{26.67\ \text{kN (Tension)}}$$

5. Final answers (neatly tabulated)

Support reactions

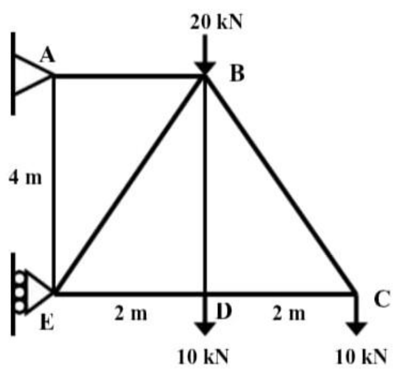
- $R_A = 20\text{kN} \uparrow$
- $R_B = 20\text{kN} \uparrow$

Forces in members

Member	**Force (kN)**	**Nature**
AD	33.33	Compression
DC	33.33	Compression
AE	26.67	Tension
EC	26.67	Tension

Member	Force (kN)	Nature
DE (King post)	40	

2.Determine the support reactions and force in AB, BE and DE members by using Method of Section.



Solution:

Step 1: Support reactions

Vertical equilibrium of whole truss

$$\sum V = 0$$
$$R_A = 20 + 10 + 10 = \boxed{40\text{ kN (upward)}}$$

Horizontal equilibrium

Let horizontal reaction at E = R_E

Take moments about A:

$$R_E \times 4 = (10 \times 2) + (10 \times 4)$$
$$R_E \times 4 = 20 + 40 = 60$$
$$\boxed{R_E = 15\text{ kN (to the right)}}$$

Step 2: Section cut

Cut the truss through members:

- AB
- BE
- DE

This section passes through only three unknown forces, which is ideal.

Consider the left portion of the truss after cutting.

Step 3: Geometry of diagonal BE

Horizontal = 2 m
Vertical = 4 m

$$\text{Length of BE} = \sqrt{2^2 + 4^2} = 4.47 \text{ m}$$
$$\sin\ \theta = \frac{4}{4.47} = 0.894$$
$$\cos\ \theta = \frac{2}{4.47} = 0.447$$

Step 4: Find force in member AB

Take moments about point E
(Forces in BE and DE pass-through E, so eliminated)

$$\sum M_E = 0$$
$$AB \times 4 = 0$$
$$\boxed{AB = 0}$$

Step 5: Find force in member BE

Take moments about point A
(Forces in AB and reaction at A eliminated)

$$BE \sin\ \theta \times 2 = 15 \times 4$$
$$BE \times 0.894 \times 2 = 60$$
$$BE = \frac{60}{1.788} = \boxed{44.7 \text{ kN}}$$

Step 6: Find force in member DE

Apply horizontal equilibrium of left section:

$$\sum H = 0$$
$$DE + 15 = BE \cos\ \theta$$
$$DE = (44.7 \times 0.447) - 15$$
$$DE = 20 - 15 = \boxed{5 \text{ kN}}$$

Member	Force (kN)	Nature
AB	0	Zero-force
BE	44.7 kN	Tension
DE	5 kN	Compression