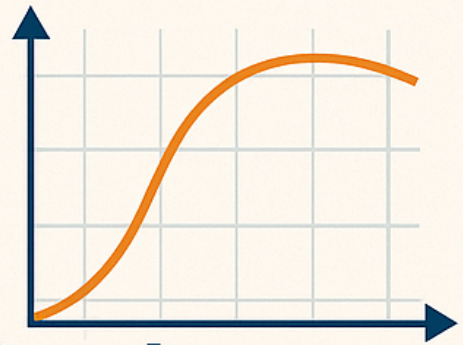


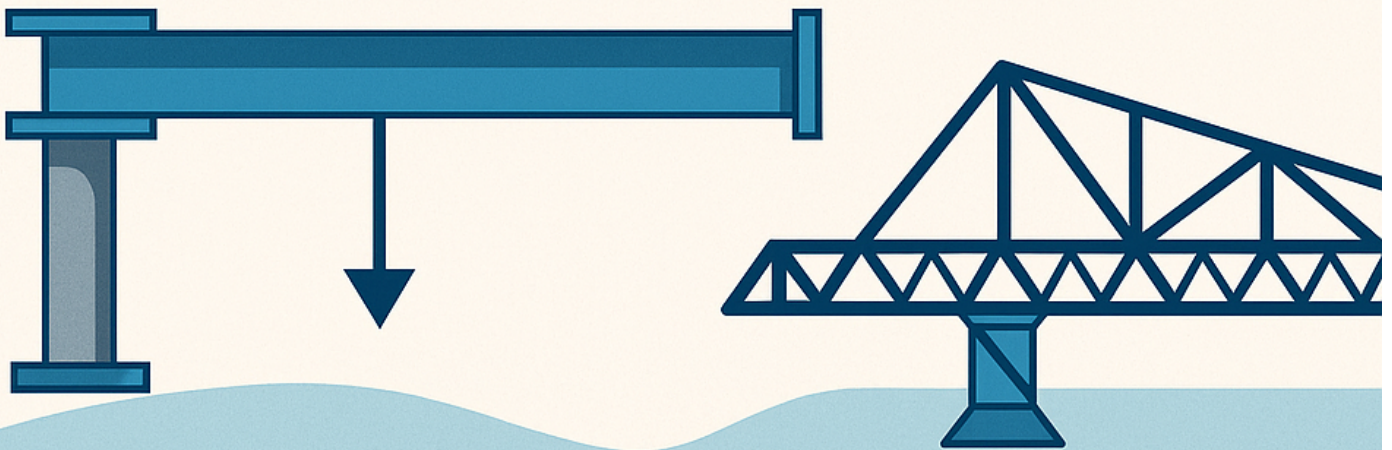
GOVERNMENT POLYTECHNIC, BHUBANESWAR

DEPARTMENT OF CIVIL ENGINEERING



Question Bank

MECHANICS OF MATERIALS



3rd Semester

2 Marks

Q.1. Define Centre of Gravity.

Ans- Centre of Gravity (C.G.):

The centre of gravity is the point at which the entire weight of a body may be considered to act, irrespective of its position. It depends on both the shape and the distribution of mass (or weight) of the body.

Q.2. Calculate the moment of inertia for the given figure about x-x axis.

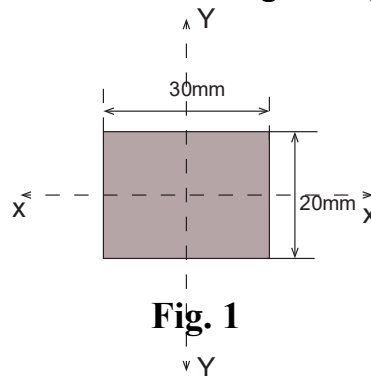


Fig. 1

Ans- The moment of inertia about x-x axis is given by $\frac{bd^3}{12} = \frac{30 \times 20^3}{12} = 20000 \text{ mm}^4$

Q.3. State parallel axis theorem.

Ans- Parallel axis theorem: It states that the moment of inertia of a rigid body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axis.

Q.4. Define the term centroid.

Ans- Centroid: The centroid is the geometric centre of a plane figure or a solid body. It is the point at which the entire area (for plane figures) or volume (for solids) of the shape can be considered to be concentrated. The centroid depends only on the shape and dimensions of the body, not on its mass or material.

Q.5. What is a built up of section?

Ans- A built-up section is a structural section that is made by joining two or more simple sections (like plates, angles, channels, or I-sections) together to form a single stronger composite section. It is used when a single rolled section is not sufficient to carry the required load or to provide the needed stiffness.

Q.6. What is moment of inertia?

Ans- The moment of inertia of an area about a given axis is the measure of the distribution of that area with respect to the axis. It indicates how the area is spread out from the axis.

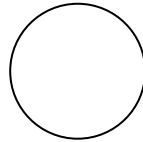
Q.7. State any 3 types of symmetrical section with diagram.

Ans-

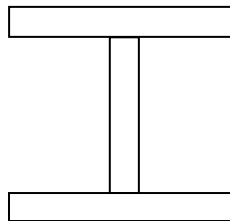
1. Rectangular section



2. Circular section



3. I section



Q.8. What is perpendicular axis theorem?

Ans- The moment of inertia of an area about any axis perpendicular to the area and passing through the centroid is equal to sum of the moments of inertia about two mutually perpendicular axes through the centroid.

Q.9. Define Radius of gyration?

Ans- The radius of gyration of an area about a given axis is the distance from the axis at which the entire area may be assumed to be concentrated so that its moment of inertia remains the same as that of the actual area.

Q.10. Define section modulus?

Ans- The section modulus (Z) is defined as the ratio of the moment of inertia (I) of the section about the neutral axis to the distance (y) from the neutral axis to the outermost fiber of the section.

$$Z = \frac{I}{y_{\max}}$$

5 Marks

Q.1. State theorem of parallel axis, write the expression and meaning of each term in the expression?

Ans- The **Theorem of Parallel Axis** states that if the moment of inertia of a plane area about an axis through its center of gravity is denoted by I_G then the moment of inertia of the area about any other axis AB, which is parallel to the first axis and at a distance h from the center of gravity, is given by the expression:

Expression:

$$I_{AB} = I_G + ah^2$$

Meaning of each term:

I_{AB} = Moment of inertia of the area about the axis AB

.

I_G = Moment of inertia of the area about an axis through its center of gravity.

a = Area of the section.

h = The perpendicular distance between the center of gravity of the section and the axis AB.

Q.2. Find the C.G of the section below.

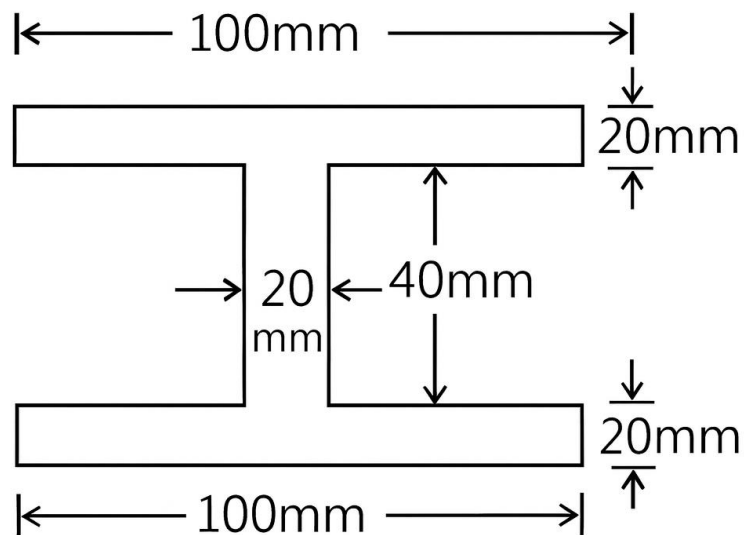


Fig. 2

Ans - (i) Bottom flange:

$$a_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$y_1 = 20 / 2 = 10 \text{ mm.}$$

(ii) Web:

$$a_2 = 40 \times 20 = 800 \text{ mm}^2.$$

$$y_2 = 20 + 40 / 2 = 40 \text{ mm.}$$

(iii) Top flange:

$$a_3 = 100 \times 20 = 2000 \text{ mm}^2.$$

$$y_3 = 20 + 40 + 20 / 2 = 70 \text{ mm.}$$

The distance between CG of the section and bottom of the flange:

$$\begin{aligned}
 \bar{y} &= (a_1 y_1 + a_2 y_2 + a_3 y_3) / (a_1 + a_2 + a_3) \\
 &= (2000 \times 10 + 800 \times 40 + 2000 \times 70) / (2000 + 800 + 2000) \\
 &= (20000 + 32000 + 140000) / 4800 \\
 &= 192000 / 4800 = 40 \text{ mm}
 \end{aligned}$$

Q.3. State theorem of perpendicular axis, write the expression and meaning of each term in the expression?

Ans – The Perpendicular Axis Theorem states that —

The moment of inertia of a plane lamina about an axis perpendicular to its plane (Z-axis) is equal to the sum of the moments of inertia about two mutually perpendicular axes (X and Y) lying in the plane and intersecting at the same point.

Mathematical Expression:

$$I_z = I_x + I_y$$

Where:

I_z = Moment of inertia about the axis perpendicular to the plane of the lamina

I_x = Moment of inertia about the X-axis lying in the plane of the lamina

I_y = Moment of inertia about the Y-axis lying in the plane of the lamina

Q.4. Determine the moment of inertia of a triangular area about its base whose length is b and height of the vertex from the base is h .

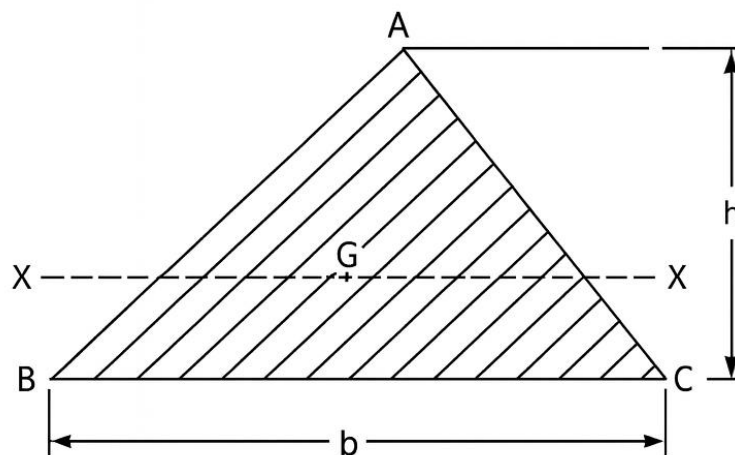


Fig. 3

Ans- Let G = C.G. of the triangular area ABC ,

b = length of base BC , and

h = altitude of the triangle from BC .

Also, let

I_{BC} = required M.I. of the triangular area ABC about its base BC .

Then, according to the theorem of parallel axes,

$I_{BC} = I_G + A h_1^2$, where

$I_G = I_{XX} = \text{M.I. of the triangle ABC about its centroidal axis XX parallel to the base BC}$

$$= \frac{1}{36}bh^3,$$

$A = \text{area of the triangle ABC} = \frac{1}{2}bh$, and

$h_1 = \text{distance of C.G. (G) of the triangle ABC from the base BC}$

$$= \frac{1}{3}h$$

$$\begin{aligned} \therefore I_{BC} &= \frac{1}{36}bh^3 + \frac{1}{2}bh \times \left(\frac{1}{3}h\right)^2 \\ &= \frac{1}{36}bh^3 + \frac{1}{18}bh^3 \\ &= \frac{1}{12}bh^3 \end{aligned}$$

Q.5. Find the centre of gravity of a T section with flange 150mmX50mm and web as 150mmX50mm about X-X and Y-Y axis?

Ans-

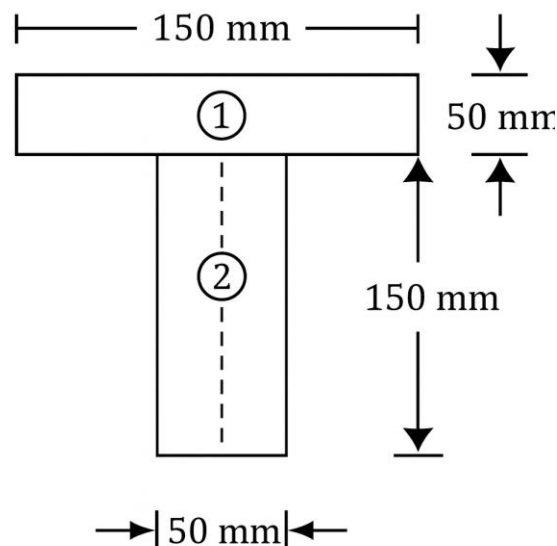


Fig. 4

Divide the T-section into two rectangles: the web and the flange. We will establish a reference axis at the bottom of the web to calculate the centroid's vertical position (\bar{y})

- Web (Part 1): A rectangle with dimensions 50 mm (width) and 150 mm (height).

UNIT I (CENTRE OF GRAVITY AND MOMENT OF INERTIA)

- Area: $A_1 = 50 \times 150 = 7500 \text{ mm}^2$
- Centroid from the bottom reference axis: $y_1 = 150/2 = 75 \text{ mm}$
- Flange (Part 2): A rectangle with dimensions 150 mm (width) and 50 mm (height).
 - Area: $A_2 = 150 \times 50 = 7500 \text{ mm}^2$
 - Centroid from the bottom reference axis: $y_2 = 150 + 50/2 = 175 \text{ mm}$

The total area is $A = A_1 + A_2 = 7500 + 7500 = 15000 \text{ mm}^2$.

Calculate the centroidal distance \bar{y}

$$\bar{y} = (A_1 y_1 + A_2 y_2) / (A_1 + A_2)$$

$$\begin{aligned}\bar{y} &= ((7500)(75) + (7500)(175)) / 15000 \\ &= (562500 + 1312500) / 15000 \\ &= 1875000 / 15000 \\ &= 125 \text{ mm}\end{aligned}$$

The center of gravity is located **125 mm** from the bottom of the web.

10 Marks

Q.1. A compound section is built-up by welding two plates 200mm x 15mm on two steel beams ISJB 200 placed symmetrically side by side as shown in the figure below. What is the moment of inertia of the compound section about an axis passing through its center of gravity and parallel to X-X axis? Take I_{xx} for the ISJB section as $7.870 \times 10^6 \text{ mm}^4$.

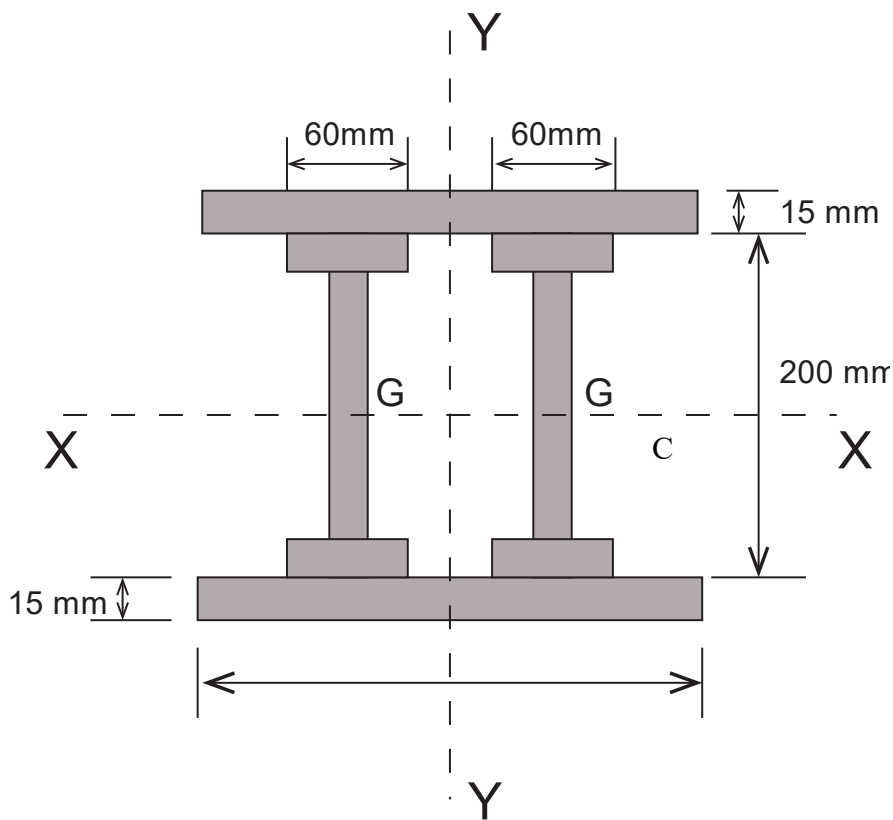


Fig. 5

Ans – Given:

Size of two plates = 200mm x 15mm and moment of inertia of ISJB 200 section about X-X axis = $7.807 \times 10^6 \text{ mm}^4$.

From the geometry of the compound section, we find that it is symmetrical about both the X-X and Y-Y axis. Therefore, centre of gravity of the section will lie at G i.e., centre of gravity of the beam sections.

We know that moment of inertia of one plate section about an axis passing through its centre of gravity and parallel to the X – X axis,

$$I_G = \frac{200 \times 15^3}{12} = 0.056 \times 10^6 \text{ mm}^4$$

And distance between the centre of gravity of the plate section and X-X axis,

UNIT I (CENTRE OF GRAVITY AND MOMENT OF INERTIA)

$$h = 100 + 15/2 = 107.5\text{mm}$$

Therefore, moment of inertia of the plate section about the X-X axis
 $= I_G + ah^2 = (0.056 \times 10^6) + (200 \times 15) \times 107.5^2 = 34.725 \times 10^6 \text{mm}^4$

and moment of inertia of the compound section about X-X axis,
 $I_{xx} = \text{Moment of inertia of 2 ISJB sections} + \text{Moment of inertia of 2 plate sections}$
 $= [2 \times (7.807 \times 10^6) + 2 \times (34.725 \times 10^6)] = 85.064 \times 10^6 \text{ mm}^4$

Q.2. Find moment of inertia of the I section about two centroidal axes at right angle to each other. with the following dimensions.

Flanges – 15cm x 1cm

Web – 28 cm x 1cm

Overall depth of the section – 30 cm.

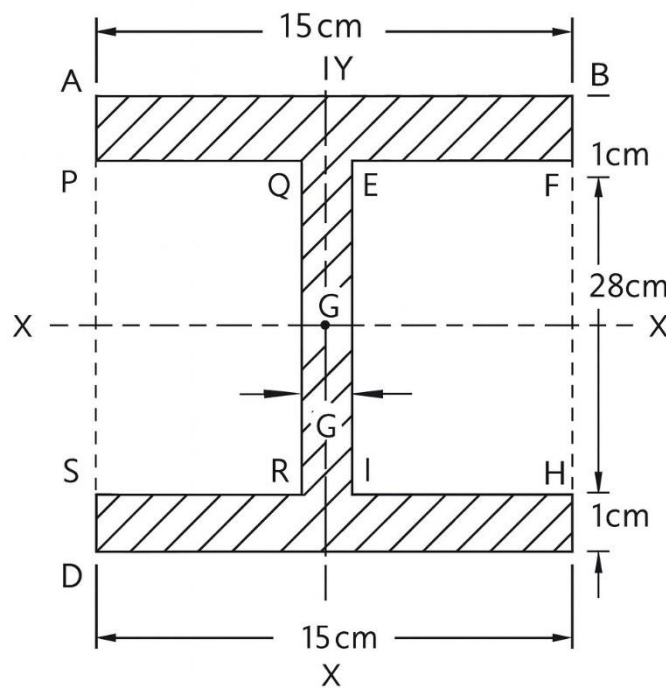


Fig. 6

Ans – Let,

G = C.G. of the whole I-section

I_{xx} = required M.I. of the I-section about the centroidal axis XX, and

I_{yy} = required M.I. of the I-section about the centroidal axis YY.

M.I. About XX-axis

$$I_{xx} = I_1 - I_2 - I_3,$$

where,

I_1 = M.I. of the rectangle ABCD about XX-axis

$$= (1/12) \times 15 \times (30)^3 \text{ cm}^4 = 33750 \text{ cm}^4$$

UNIT I (CENTRE OF GRAVITY AND MOMENT OF INERTIA)

$$I_2 = \text{M.I. of the rectangle EFHI about XX-axis} \\ = (1 / 12) \times 7 \times (28)^3 \text{ cm}^4 = 12805.33 \text{ cm}^4$$

$$I_3 = \text{M.I. of the rectangle PQRS about XX-axis} \\ = (1 / 12) \times 7 \times (28)^3 \text{ cm}^4 = 12805.33 \text{ cm}^4$$

$$\therefore I_{xx} = 33750 - 12805.33 - 12805.33 \\ = 8139.34 \text{ cm}^4$$

M.I. About YY-axis

$$I_{yy} = I_1' + I_2' + I_3',$$

where,

$$I_1' = \text{M.I. of the top flange about YY-axis} \\ = (1 / 12) \times 1 \times (15)^3 \text{ cm}^4 = 281.25 \text{ cm}^4$$

$$I_2' = \text{M.I. of the bottom flange about YY-axis} \\ = (1 / 12) \times 1 \times (15)^3 \text{ cm}^4 = 281.25 \text{ cm}^4$$

$$I_3' = \text{M.I. of the web about YY-axis} \\ = (1 / 12) \times 28 \times 1^3 \text{ cm}^4 = 2.33 \text{ cm}^4$$

$$\therefore I_{yy} = 281.25 + 281.25 + 2.33 = 564.83 \text{ cm}^4$$

2 MARKS

Q.1) Write the relationship between E, G, and K.

Ans – The relationship between elastic constants is given by:

$$E = 2G(1 + \mu) = 3K(1 - 2\mu)$$

where,

E = Modulus of Elasticity

G = Modulus of Rigidity

K = Bulk Modulus

μ = Poisson's Ratio

Q.2) What are Principal Stresses and Principal Planes

Ans – Principal stresses are the **maximum and minimum normal stresses** acting on planes where **shear stress is zero**.

The planes on which these stresses act are called **principal planes**.

Q.3) Define Normal Stress and Shear Stress on an inclined plane.

Ans – Normal Stress (σ_n):

It is the component of stress that acts perpendicular to an inclined plane.

It tends to either pull apart (tension) or compress (compression) the material.

Shear Stress (τ):

It is the component of stress that acts tangentially to an inclined plane.

It tends to cause one layer of the material to slide over the other.

Q.4) What is the significance of Mohr's Circle?

Ans – Mohr's Circle is a **graphical method** used to determine the **magnitude and orientation** of normal and shear stresses on any inclined plane, and to locate **principal stresses and planes**.

Q.5) What do you mean by Major and Minor Principal Stresses?

Ans – The **major principal stress** is the **maximum normal stress**, and the **minor principal stress** is the **minimum normal stress** acting on mutually perpendicular principal planes.

Q.6) Write the formula for Normal and Shear Stress on an inclined plane.

Ans – Normal stress: $\sigma_n = (\sigma_x + \sigma_y)/2 + (\sigma_x - \sigma_y)/2 \times \cos(2\theta) + \tau_{xy} \times \sin(2\theta)$

Shear stress: $\tau = -(\sigma_x - \sigma_y)/2 \times \sin(2\theta) + \tau_{xy} \times \cos(2\theta)$

Q.7) What is meant by Biaxial and Triaxial Stress?

Ans – Biaxial Stress:

When a material is subjected to **two different normal stresses** acting in **two mutually perpendicular directions**, it is said to be under *biaxial stress*.

Triaxial Stress:

When a material is subjected to **three normal stresses** acting **mutually perpendicular** to each other, it is said to be under *triaxial stress*.

Q.8) Define Volumetric Strain.

Ans – Volumetric strain is the **ratio of change in volume to the original volume** of a body under stress.

Formula: $\epsilon_v = \Delta V / V$

Q.9) Explain Temperature Stress in a simple bar.

Ans – When a bar is **prevented from expanding or contracting** due to a temperature change, **thermal stress** develops.

Formula: $\sigma_T = \alpha \times E \times \Delta T$

where α = coefficient of thermal expansion, E = modulus of elasticity, ΔT = temperature change.

Q.10) What is the Yield Point in a stress–strain curve?

Ans – The yield point is the stage at which a material **begins to deform permanently** without any increase in load — marking the **end of elastic behaviour**.

5 MARKS

Q.1) At a point in a strained material, the normal stress on the x-plane is 80 MPa (tensile), the normal stress on the y-plane is 40 MPa (compressive), and the shear stress acting on the plane is 30 MPa. Using the Mohr's Circle method, determine the principal stresses and their magnitudes, the maximum shear stress and the corresponding normal stress on that plane, and the inclination of the principal planes with respect to the x-axis. Show all necessary steps and draw the Mohr's Circle neatly.

Ans – Given: $\sigma_x = +80$ MPa $\sigma_y = -40$ MPa $\tau_{xy} = 30$ MPa

Centre of Mohr's Circle (C)

$$\begin{aligned} C &= (\sigma_x + \sigma_y) / 2 \\ &= (80 - 40) / 2 \\ &= 20 \text{ MPa} \end{aligned}$$

Radius of Mohr's Circle (R):

$$\begin{aligned} R &= \sqrt{[(\sigma_x - \sigma_y) / 2]^2 + (\tau_{xy})^2} \\ &= \sqrt{[(80 - (-40)) / 2]^2 + (30)^2} \\ &= \sqrt{(60)^2 + (30)^2} \\ &= 67.1 \text{ MPa} \end{aligned}$$

Principal Stresses:

$$\sigma_{1,2} = C \pm R$$

$$\sigma_1 = 20 + 67.1 = 87.1 \text{ MPa}$$

$$\sigma_2 = 20 - 67.1 = -47.1 \text{ MPa}$$

Maximum Shear Stress:

$$\tau_{\max} = R = 67.1 \text{ MPa}$$

Normal Stress on Plane of Maximum Shear:

$$\sigma_{\text{avg}} = C = 20 \text{ MPa}$$

Angle of Principal Planes:

$$\begin{aligned} \tan 2\theta_p &= (2 \tau_{xy}) / (\sigma_x - \sigma_y) \\ &= (2 \times 30) / (80 - (-40)) \\ &= 60 / 120 = 0.5 \end{aligned}$$

UNIT II – SIMPLE STRESSES AND STRAINS

$$2\theta_p = 26.565^\circ$$

$$\theta_p = 13.28^\circ$$

Principal Stresses, $\sigma_1 = 87.1 \text{ MPa}$, $\sigma_2 = -47.1 \text{ MPa}$

Maximum Shear Stress, $\tau_{\max} = 67.1 \text{ MPa}$

Normal Stress on Shear Plane = 20 MPa

Inclination of Principal Plane = 13.28° from x-axis

Q.2) Draw and explain the Stress–Strain Curve for a Mild Steel Bar under Tension.

Ans: Important Points on the Stress–Strain Curve:

A – Proportional Limit:

- i. Up to this point, stress is directly proportional to strain.
- ii. The material obeys **Hooke's Law** ($\sigma \propto \epsilon$).
- iii. The curve is a straight line (linear region).
- iv. The slope of this line represents the **Modulus of Elasticity (E)**.

B – Elastic Limit (or Limit of Elasticity):

- i. Beyond this point, stress and strain are no longer exactly proportional.
- ii. However, the material still returns to its original shape after the load is removed.
- iii. The elastic behavior ends here.

C – Upper Yield Point:

- i. The material starts to yield — large strains occur with little or no increase in stress.
- ii. This is the **upper yield stress** value where the material begins plastic deformation.
- iii. Internal molecular slip begins.

D – Lower Yield Point:

- i. After the upper yield point, the stress slightly drops to the **lower yield point** and remains nearly constant over a range of strain.
- ii. Material continues to elongate without an increase in load.
- iii. Marks the start of **plastic deformation**.

E – Strain Hardening Point:

- i. Beyond yielding, the metal becomes stronger due to **strain hardening** (dislocation movement).

UNIT II – SIMPLE STRESSES AND STRAINS

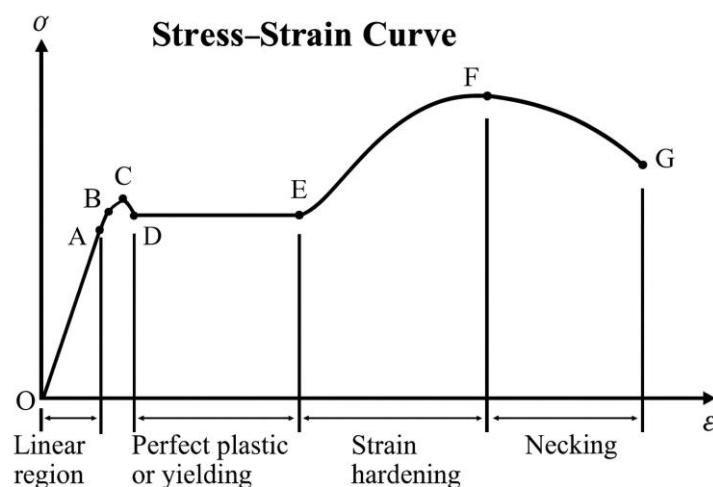
- ii. Additional stress is required to cause further deformation.
- iii. The curve rises again up to the **ultimate stress point**.

F – Ultimate Stress Point:

- i. The material can bear the **maximum stress** at this point, known as the **Ultimate Tensile Strength (UTS)**.
- ii. Beyond this point, **necking** starts — reduction in cross-sectional area of the specimen.

G – Fracture Point (Breaking Point):

- i. The specimen finally breaks at this point.
- ii. Stress appears to drop because the load-carrying cross-section has significantly reduced.



Q3) Explain the different types of stresses — Normal, Direct, Bending and Shear. Also state the nature of stresses, i.e., Tensile and Compressive stresses. (5 marks)

Ans: Stress is the internal resistance offered by a material per unit area when subjected to an external load.

1. Normal Stress (σ):

When the load acts perpendicular (normal) to the cross-section, the stress developed is called Normal Stress.

Formula: $\sigma = P / A$

If the load tends to elongate the body, it is *Tensile Stress*, and if it shortens the body, it is *Compressive Stress*.

2. Direct Stress:

When a body is subjected to an axial load acting along its longitudinal axis

(like a bar under tension or compression), the stress produced is called Direct Stress.

It is uniform over the entire cross-section.

3. Bending Stress (σ_b):

When a beam or member bends due to an external moment or transverse load, the top fibres experience compression and the bottom fibres experience tension.

Formula: $\sigma_b = M \times y / I$

where,

M = Bending moment

y = Distance from neutral axis

I = Moment of inertia of the section

4. Shear Stress (τ):

When the load acts tangentially or parallel to the surface, the stress developed is called Shear Stress.

Formula: $\tau = V / A$

where,

V = Shear force

A = Area resisting shear

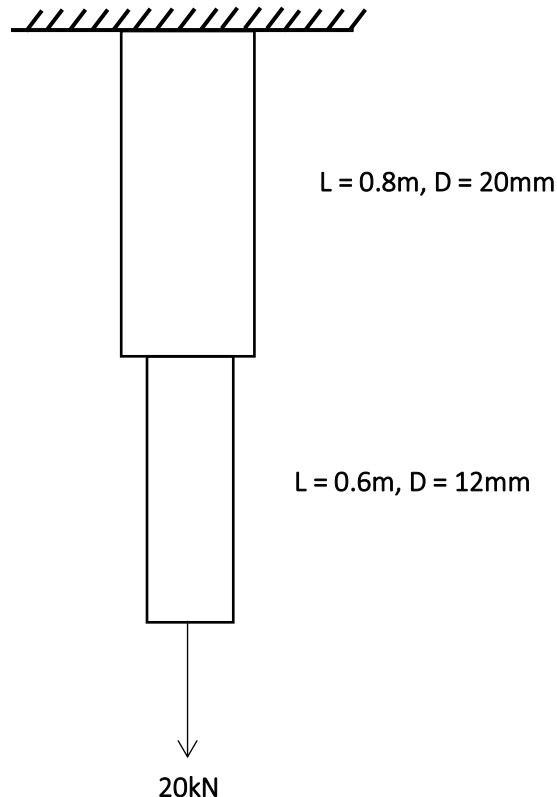
Nature of Stresses:

- Tensile Stress: Acts to elongate or stretch the material.
- Compressive Stress: Acts to shorten or compress the material.

Hence, depending on the direction and type of external load, different stresses develop in a member which govern its strength and deformation behavior.

UNIT II – SIMPLE STRESSES AND STRAINS

Q.4) A vertical composite bar consists of two steel segments in series. The upper segment is 0.8 m long and 20 mm in diameter, while the lower segment is 0.6 m long and 12 mm in diameter. The bar is fixed at the top and carries a uniaxial tensile load of 20 kN at its lower end. Take $E = 200 \text{ GPa}$. Neglect the self-weight of the bar. Calculate the total elongation of the bar.



Ans – Given:

$$L_1 = 0.8 \text{ m}, \quad d_1 = 20 \text{ mm} = 0.020 \text{ m}$$

$$L_2 = 0.6 \text{ m}, \quad d_2 = 12 \text{ mm} = 0.012 \text{ m}$$

$$P = 20 \text{ kN} = 20,000 \text{ N}$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$$

Required:

$$\text{Total elongation, } \Delta = \Delta_1 + \Delta_2$$

Formula:

$$\Delta_i = (P \times L_i) / (A_i \times E), \quad \text{where } A_i = (\pi \times d_i^2) / 4$$

Step 1: Calculate Areas

$$A_1 = (\pi \times 0.020^2) / 4 = 3.1416 \times 10^{-4} \text{ m}^2$$

$$A_2 = (\pi \times 0.012^2) / 4 = 1.1310 \times 10^{-4} \text{ m}^2$$

Step 2: Calculate Elongations

$$\begin{aligned}\Delta_1 &= (20,000 \times 0.8) / (3.1416 \times 10^{-4} \times 200 \times 10^9) \\ &= 2.546 \times 10^{-4} \text{ m} = 0.2546 \text{ mm}\end{aligned}$$

$$\begin{aligned}\Delta_2 &= (20,000 \times 0.6) / (1.1310 \times 10^{-4} \times 200 \times 10^9) \\ &= 5.305 \times 10^{-4} \text{ m} = 0.5305 \text{ mm}\end{aligned}$$

Step 3: Total Elongation

$$\begin{aligned}\Delta &= \Delta_1 + \Delta_2 \\ &= 0.2546 \text{ mm} + 0.5305 \text{ mm} \\ &= 0.7851 \text{ mm}\end{aligned}$$

Q.5) A copper rod of 2 m length and 25 mm diameter is fixed rigidly at both ends. If the temperature of the rod is increased by 50°C, determine: The thermal strain and thermal stress developed in the rod (if expansion is fully prevented)

Take:

Coefficient of linear expansion, $\alpha = 17 \times 10^{-6} / ^\circ\text{C}$

Modulus of elasticity, $E = 110 \text{ GPa}$.

Ans –

Given:

Length, $L = 2 \text{ m} = 2000 \text{ mm}$

Diameter, $d = 25 \text{ mm}$

Rise in temperature, $\Delta T = 50^\circ\text{C}$

$\alpha = 17 \times 10^{-6} / ^\circ\text{C}$

$E = 110 \text{ GPa} = 110 \times 10^3 \text{ N/mm}^2$

Ans: (a) Thermal Strain:

Thermal strain is the strain produced due to change in temperature,

$$\begin{aligned}\epsilon_t &= \alpha \times \Delta T \\ &= (17 \times 10^{-6}) \times 50 \\ &= 850 \times 10^{-6} \\ &= 0.00085\end{aligned}$$

Hence,

Thermal strain, $\epsilon_t = 0.00085$.

(b) Thermal Stress (when expansion is fully prevented):

If the bar is completely fixed and expansion is not allowed,

stress developed = $E \times \epsilon_t$

$$\sigma = E \times \alpha \times \Delta T$$

$$= (110 \times 10^3) \times (17 \times 10^{-6}) \times 50$$

$$= 93.5 \text{ N/mm}^2$$

Hence,

Thermal stress, $\sigma = 93.5 \text{ MPa}$ (compressive).

10 MARKS

Q.1) (a) Define temperature stress and temperature strain.

(b) Explain how temperature stress is developed in a bar when its expansion or contraction is prevented.

Ans –

(a) Definition of Temperature Stress and Temperature Strain:

When the temperature of a body changes, it tends to expand on heating and contract on cooling.

If this expansion or contraction is restricted, stresses are developed within the material.

These stresses are known as Temperature Stresses.

If the body were free to expand or contract, the strain produced due to change in temperature is called Temperature Strain.

Mathematically,

$$\text{Temperature Strain, } \epsilon_t = \alpha \times \Delta T$$

where,

α = coefficient of linear expansion (per °C)

ΔT = change in temperature (°C)

(b) Development of Temperature Stress in a Restrained Bar:

Consider a uniform metal bar of length L , which is fixed at both ends.

When the temperature of the bar rises by ΔT , it tries to expand freely by an amount,

$$\Delta L = \alpha \times L \times \Delta T$$

However, since the ends are rigidly fixed, this expansion is prevented.

As a result, the bar experiences compressive strain equal to the prevented expansion per unit length.

This strain produces a compressive stress in the bar given by,

$$\sigma = E \times \epsilon_t = E \times \alpha \times \Delta T$$

where,

σ = temperature stress (N/mm²)

E = modulus of elasticity (N/mm²)

α = coefficient of linear expansion (per °C)

ΔT = rise or fall in temperature (°C)

Q.2) (a) Define longitudinal strain, lateral strain, and Poisson's ratio.

(b) A steel rod 2 m long and 20 mm in diameter is subjected to a tensile stress of 100 MPa.

If Poisson's ratio (μ) = 0.3 and modulus of elasticity (E) = 200 GPa, determine the lateral strain produced in the rod.

Ans –

(a) Definitions:

Longitudinal Strain (ϵ_l):

When a member is subjected to an axial load, it undergoes a change in length. The ratio of the change in length (ΔL) to the original length (L) is called Longitudinal Strain.

$$\epsilon_l = \Delta L / L$$

Lateral Strain (ϵ_t):

When the member is stretched or compressed longitudinally, its cross-sectional dimensions change.

The ratio of change in lateral dimension (Δd) to the original lateral dimension (d) is called Lateral Strain.

$$\epsilon_t = \Delta d / d$$

Lateral strain is negative in tension (because diameter decreases) and positive in compression.

Poisson's Ratio (μ):

It is defined as the ratio of lateral strain to longitudinal strain.

$$\mu = (\text{Lateral Strain}) / (\text{Longitudinal Strain})$$

$$\mu = \epsilon_t / \epsilon_l$$

(b) Numerical Problem:

Given:

Length, $L = 2 \text{ m} = 2000 \text{ mm}$

Diameter, $d = 20 \text{ mm}$

Stress, $\sigma = 100 \text{ MPa}$

Modulus of Elasticity, $E = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$

Poisson's ratio, $\mu = 0.3$

Step 1: Calculate Longitudinal Strain

UNIT II – SIMPLE STRESSES AND STRAINS

$$\begin{aligned}\epsilon_1 &= \sigma / E \\ &= 100 / (200 \times 10^3) \\ &= 0.0005\end{aligned}$$

Step 2: Calculate Lateral Strain

$$\begin{aligned}\mu &= \epsilon_t / \epsilon_1 \\ \Rightarrow \epsilon_t &= \mu \times \epsilon_1 \\ &= 0.3 \times 0.0005 \\ &= 0.00015\end{aligned}$$

2 MARKS

Q1. Define shear force and bending moment.

Ans –

Shear Force (S.F): The algebraic sum of all vertical forces acting on one side of a section of a beam.

Bending Moment (B.M): The algebraic sum of moments of all forces acting on one side of a section about that section.

Q2. What is the relation between load, shear force, and bending moment (without derivation)?

Ans – $dV/dx = -w$ and $dM/dx = V$

Where, w = load intensity, V = shear force, M = bending moment.

Where,

w = load intensity, V = shear force, M = bending moment.

Q3. List different types of beams with one example each.

Answer:

1. Simply Supported Beam – supported at both ends.
2. Cantilever Beam – fixed at one end and free at the other.
3. Overhanging Beam – extends beyond one or both supports.
4. Fixed Beam – both ends are fixed.

Q4. What are the types of supports used in beams? Mention the reactions developed at each.

Ans – **Simple / Roller Support:** Provides one vertical reaction.

Hinged / Pinned Support: Provides one vertical and one horizontal reaction.

Fixed Support: Provides vertical, horizontal reaction and a moment.

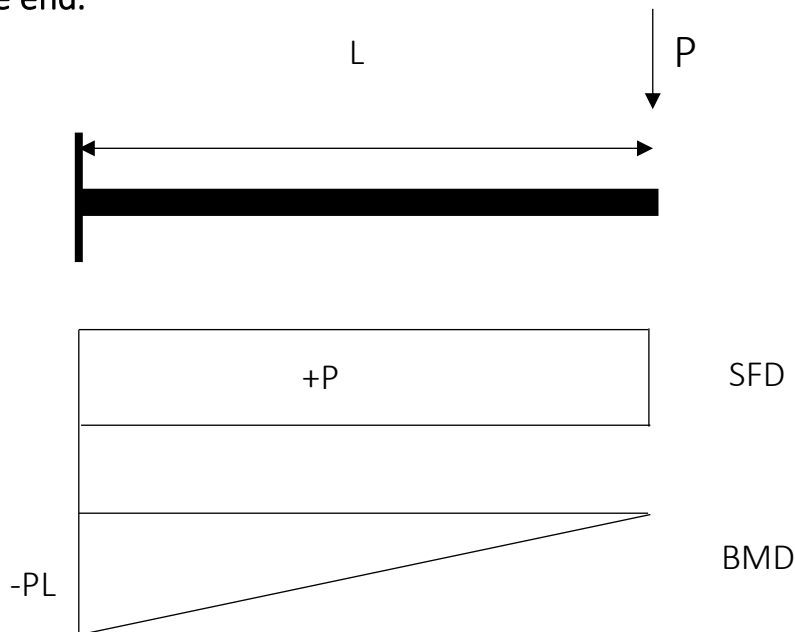
Q5. Define point of contraflexure.

Ans – A point along the beam where the bending moment changes its sign (from positive to negative or vice versa) is called the **Point of Contraflexure**.

At this point, **B.M = 0**.

Q6. Draw the S.F. and B.M. diagram for a cantilever beam carrying a point load at its free end.

Ans –



Q7. State the nature of S.F. and B.M. diagrams for a simply supported beam subjected to uniformly distributed load (U.D.L).

Ans –

S.F. Diagram: Varies linearly (inclined straight line).

B.M. Diagram: Parabolic curve, maximum at mid-span.

Q8. What is the effect of a couple acting at mid-span on a simply supported beam?

Ans –

A couple causes:

No change in shear force on either side of the section.

Sudden change (jump) in bending moment equal to the magnitude of the couple.

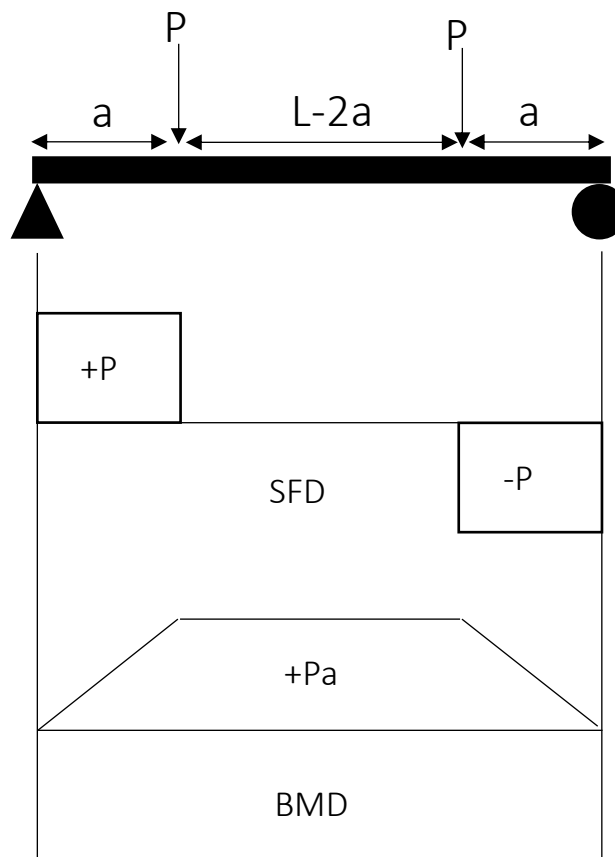
UNIT III – SIMPLE STRESSES AND STRAINS

Q9. Differentiate between point load and uniformly distributed load (U.D.L).

Ans – Point Load	Uniformly Distributed Load (U.D.L)
Acts at a single point.	Spread uniformly over a length.
Unit: kN	Unit: kN/m
Shear Force changes suddenly.	Shear Force changes linearly.

Q10. Sketch the S.F. diagram for a simply supported beam with two equal point loads symmetrically placed.

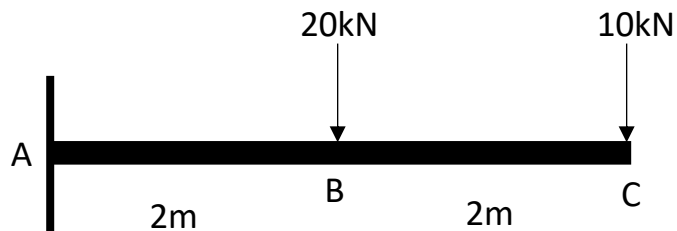
Ans –



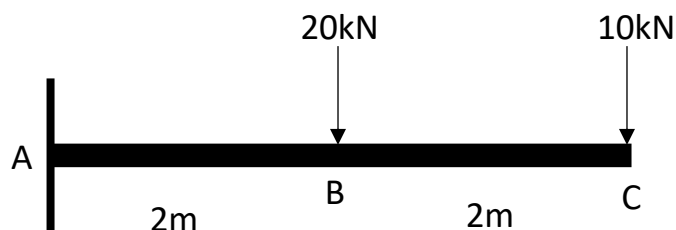
UNIT III – SIMPLE STRESSES AND STRAINS

5 MARKS

Q.1. Draw the shear force and bending moment diagram for the cantilever beam shown below.



Ans –



Shear force calculation

SF just left of C = +10kN

SF just right of B = +10kN

SF just left of B = +10kN + 20kN = +30kN

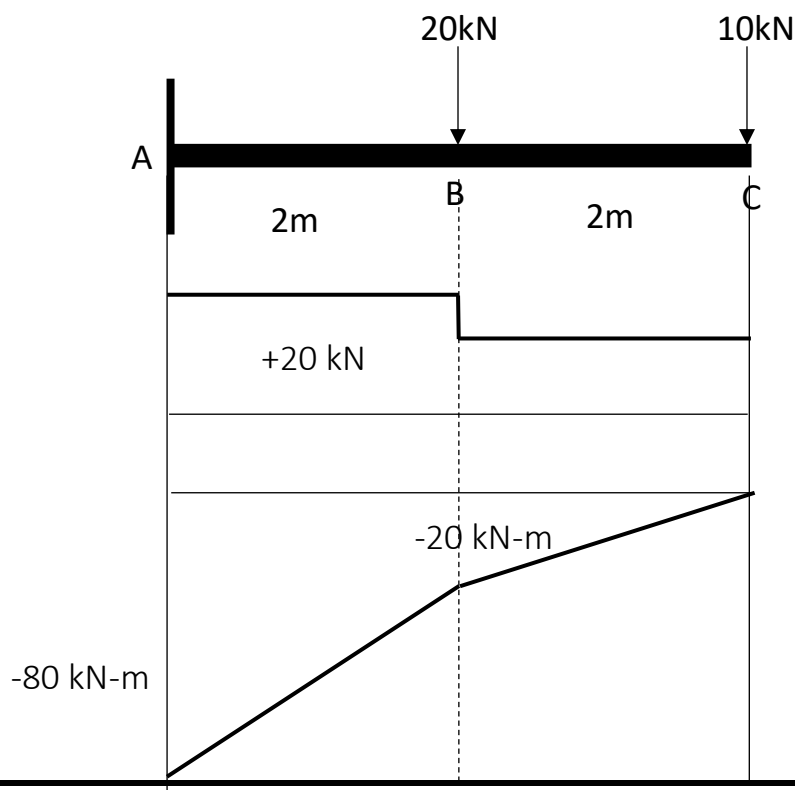
SF just left of C = +10kN + 20kN = +30kN

Bending moment calculation

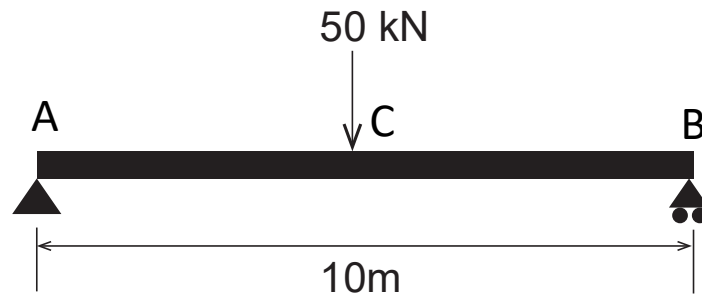
BM at C = 0

BM at B = -10 X 2 = -20kN-m

BM at A = -10 x 4 – 20 x 2 = -80kN-m



Q.2. A simply supported beam of span length 10m is subjected to a central point load of 50kN. Calculate the SF and BM at points A, B and C.



Ans – Let V_A , V_B be the support reaction. As the beam is symmetrically loaded therefore, $V_A = V_B = 25$ kN.

Shear force calculation

SF just left of B = -25 kN

SF just right of C = -25 kN

SF just left of C = -25 kN + 50 kN = +25 kN

SF just right of A = +25 kN

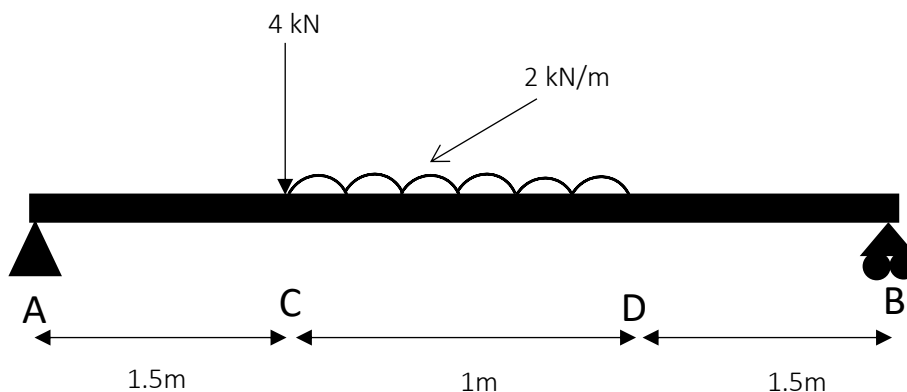
Bending moment calculation

BM at B = 0

BM at C = +25 × 5 = +125 kN-m

BM at A = 0

Q.3. A simply supported beam of 4m span is carrying loads as shown. Draw the shear force diagram & bending moment diagram.



Ans – Let V_A & V_B be the reactions at the supports

$$\sum F_y = 0$$

UNIT III – SIMPLE STRESSES AND STRAINS

$$\Rightarrow V_A + V_B - 4 \text{ kN} - 2 \text{ kN/m} \times 1 = 0$$

$$\Rightarrow V_A + V_B = 6 \text{ kN}$$

$$\sum M_a = 0$$

$$+4 \text{ kN} \times 1.5 \text{ m} + 2 \text{ kN/m} \times 1 \text{ m} \times 2 \text{ m} - V_B \times 4 = 0$$

$$\Rightarrow V_B = 10/4 = 2.5 \text{ kN}$$

$$\text{So, } V_A = 6 \text{ kN} - 2.5 \text{ kN} = 3.5 \text{ kN}$$

Shear force calculation

SF just left of B = -2.5 kN

SF just right of D = -2.5 kN

SF just right of C = -2.5 kN + 2 kN/m \times 1 m = -0.5 kN

SF just left of C = -2.5 + 2 \times 1 + 4 = 3.5 kN

SF just right of A = 3.5 kN

Bending moment calculation

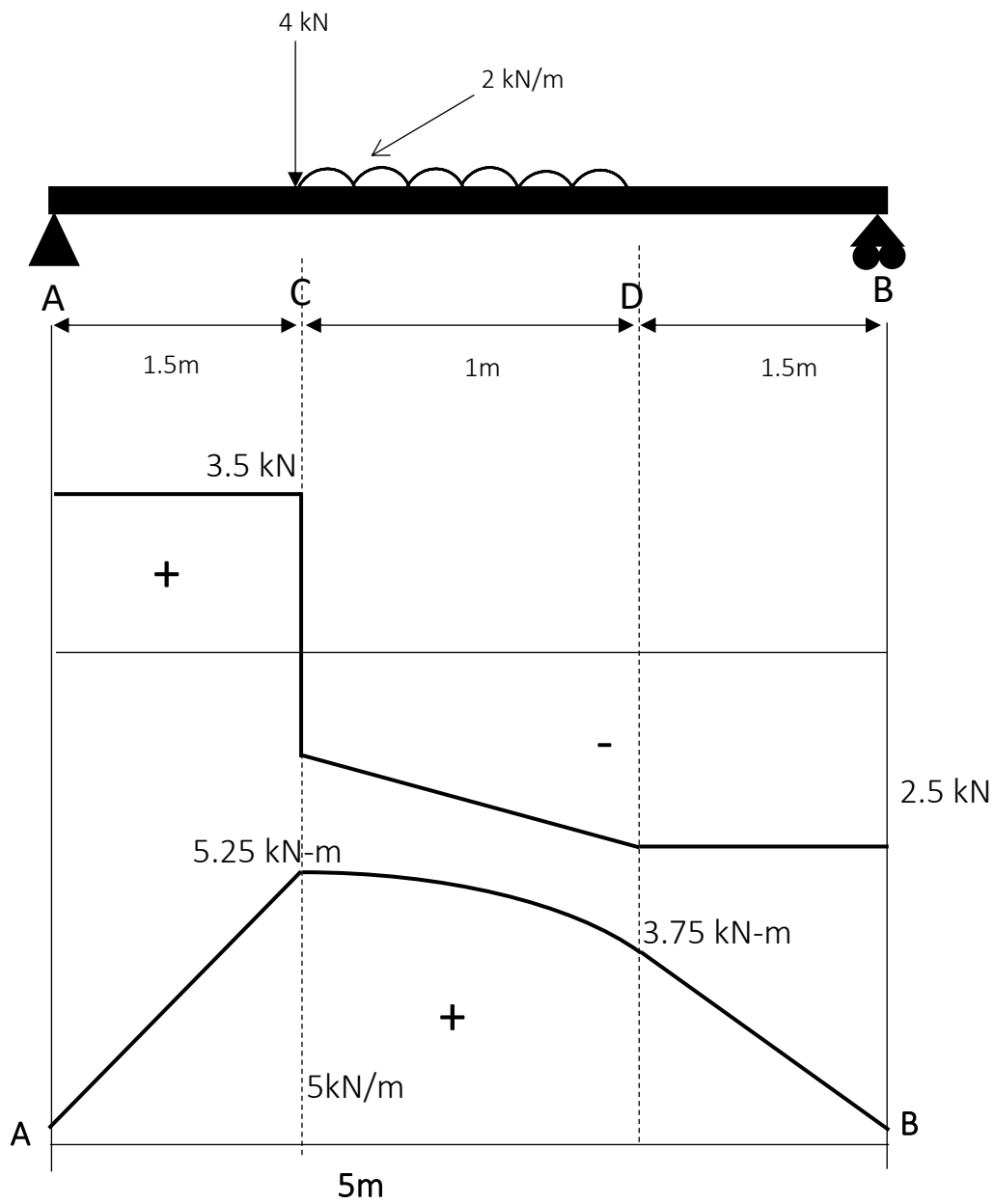
BM at B = 0

BM at D = 2.5 \times 1.5 = 3.75 kN-m

BM at C = 2.5 \times 2.5 = 6.25 kN-m

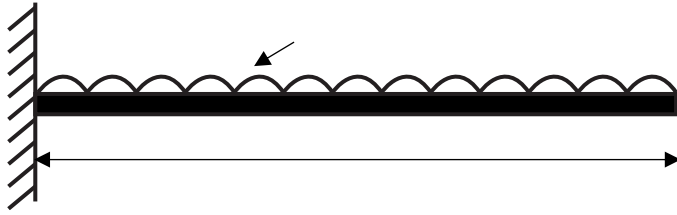
BM at A = 0

UNIT III – SIMPLE STRESSES AND STRAINS



UNIT III – SIMPLE STRESSES AND STRAINS

Q.4. Draw SFD and BMD for the beam shown below



Ans – Shear force calculation

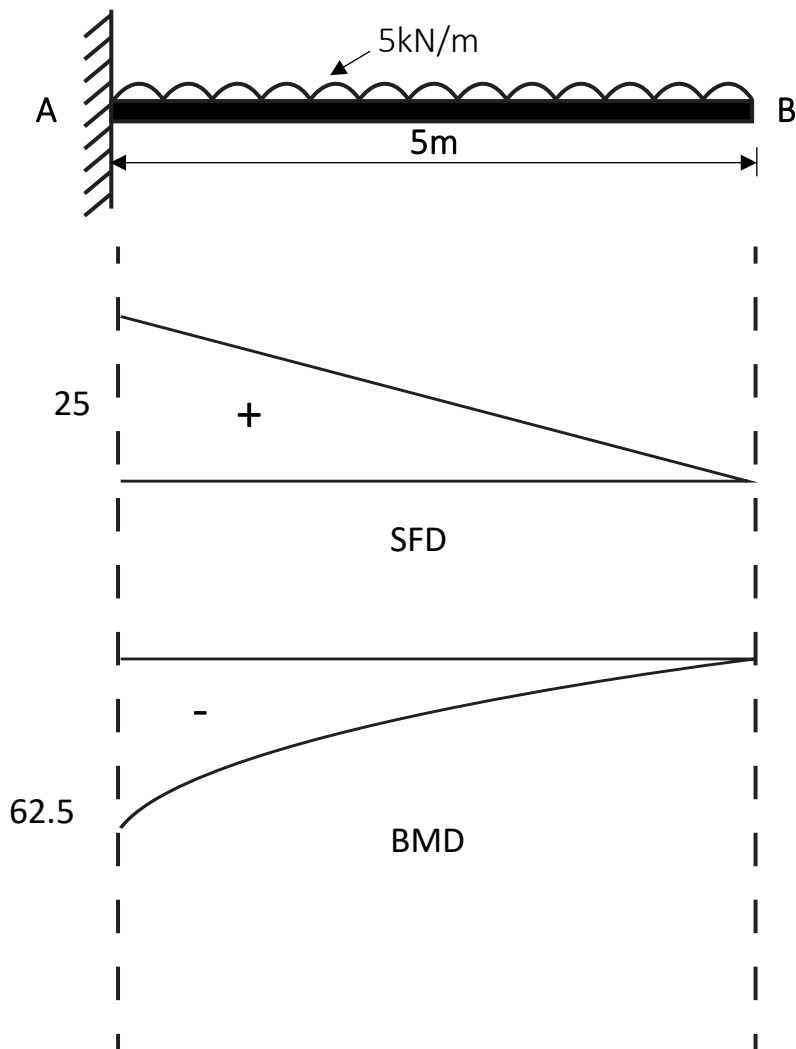
SF just left of B = 0

SF just right of A = $+5\text{ kN/m} \times 5 = +25\text{ kN}$

Bending moment calculation

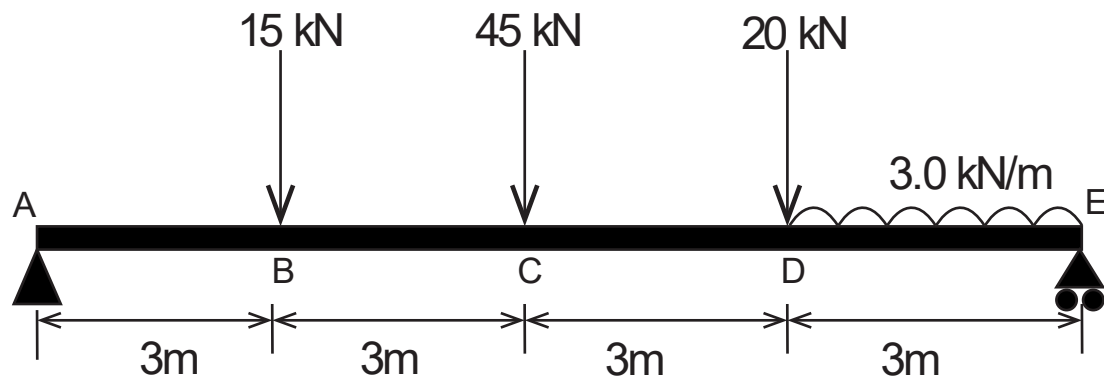
BM at B = 0

BM at A = $-5\text{ kN/m} \times 5\text{ m} \times 2.5\text{ m} = -62.5\text{ kN-m}$



UNIT III – SIMPLE STRESSES AND STRAINS

Q.5. Calculate the support reactions for the simply supported beam shown below



Ans – Let V_A & V_E be the support reactions

$$\sum F_y = 0$$

$$\Rightarrow V_A + V_E - 15 - 45 - 20 - 3 \times 3 = 0$$

$$\Rightarrow V_A + V_E = 89 \text{ kN} \quad \text{----- Eqn. 1}$$

$$\sum M_A = 0$$

$$\Rightarrow 15 \text{ kN} \times 3 + 45 \text{ kN} \times 6 + 20 \text{ kN} \times 9 + 3 \times 3 \times 10.5 - V_E \times 12 = 0$$

$$\Rightarrow 12 V_E = 589.5 \text{ kN-m}$$

$$\Rightarrow V_E = 39.875 \text{ kN}$$

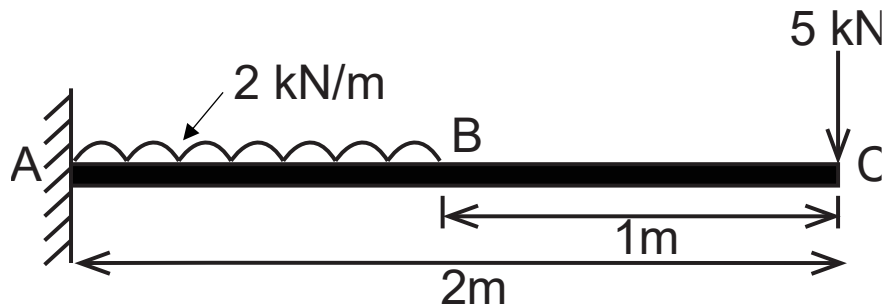
From Eqn. 1

$$V_A = 89 - 39.875 = 49.125 \text{ kN}$$

UNIT III – SIMPLE STRESSES AND STRAINS

10 MARKS

Q.1. A Cantilever beam having a span of 2m is loaded as shown. Draw SFD and BMD.



Ans –

Shear Force calculation:

SF just left of B = +5 kN

SF just right of C = +5 kN

SF just left of C = +5 kN

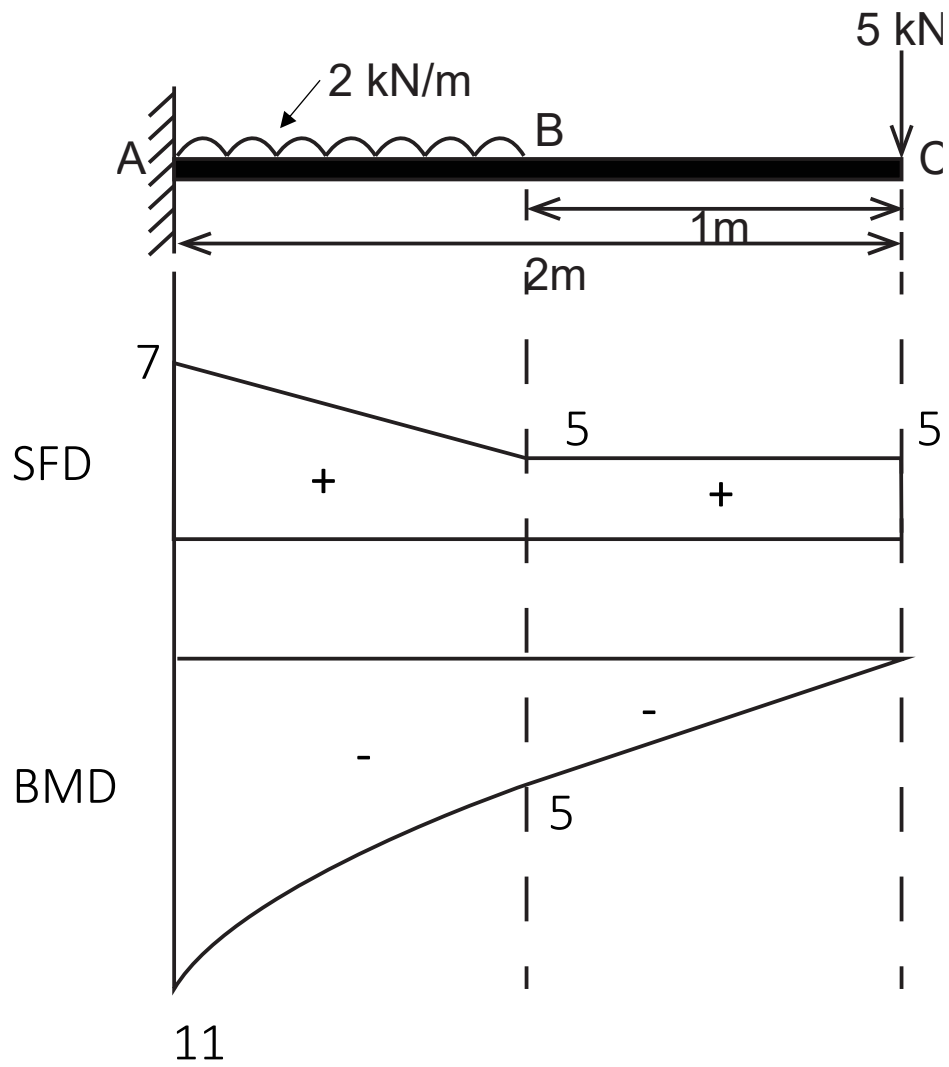
SF just right of A = +5 kN + 2kN/m \times 1m = +7 kN

Bending moment calculation:

BM at B = 0

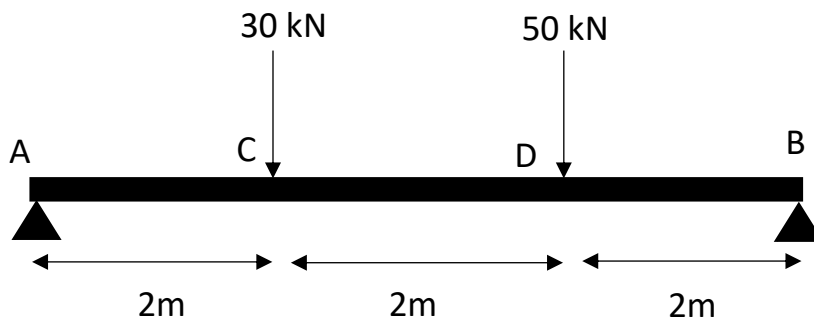
BM at C = -5 \times 1 = -5kN/m

BM at A = -5 \times 2 - 2 \times 1 \times $\frac{1}{2}$ = -11 kN-m



UNIT III – SIMPLE STRESSES AND STRAINS

Q.2. Draw the shear force and bending moment diagram for the beam shown below.



Ans – Let V_A & V_B be the reactions at the support.

$$\sum F_y = 0$$

$$V_A + V_B - 30 \text{ kN} - 50 \text{ kN} = 0$$

$$V_A + V_B = 80 \text{ kN}$$

$$\sum M_a = 0$$

$$+30 \text{ kN} \times 2 + 50 \text{ kN} \times 4 - V_B \times 6 = 0$$

$$\Rightarrow V_B = 43.33 \text{ kN}$$

From Equation (1)

$$V_A = 80 - 43.33 = 36.67 \text{ kN}$$

Shear force calculation

SF just left of B = -43.33 kN

SF just right of D = -43.33 kN

SF just left of D = -43.33 kN + 50 kN = 6.67 kN

SF just right of C = -43.33 kN + 50 kN = 6.67 kN

SF just left of C = -43.33 kN + 50 kN + 30 kN = 36.67 kN

SF just right of A = 36.67 kN

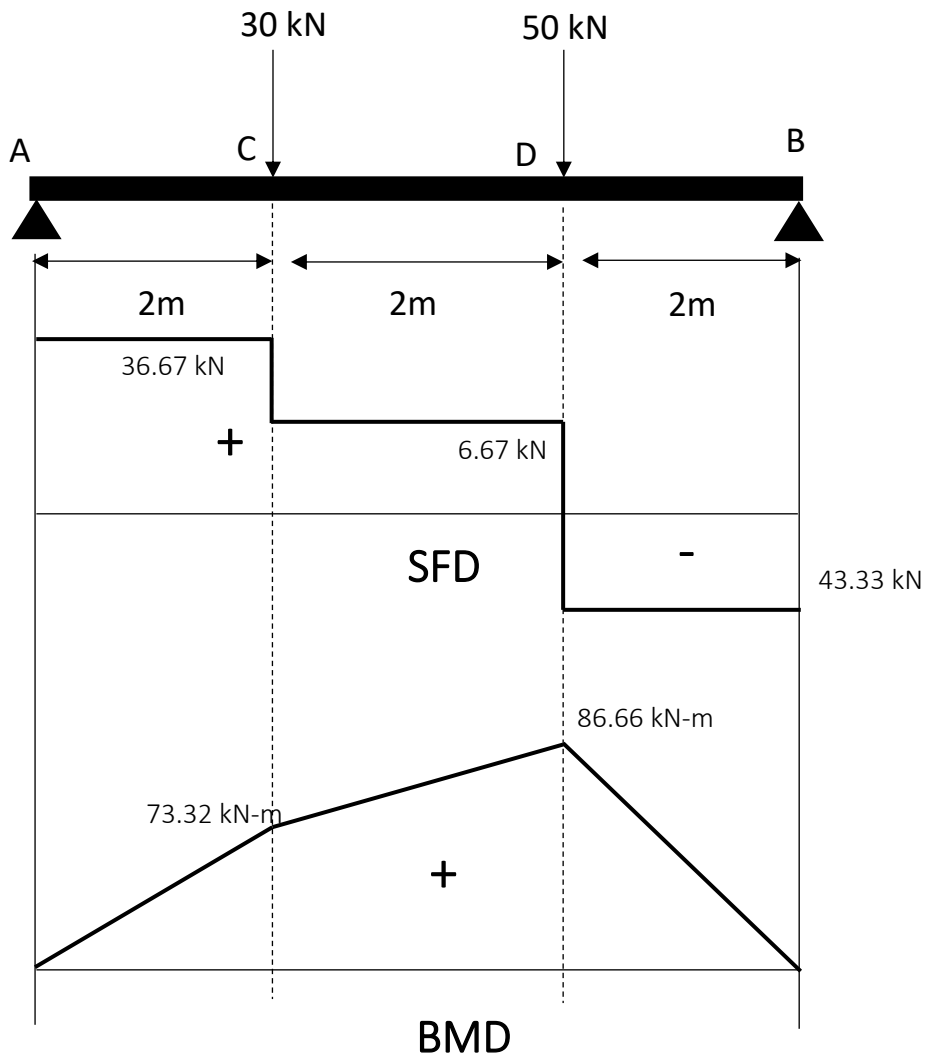
Bending moment calculation

BM at B = 0

BM at D = 43.33 x 2 = 86.66 kN-m

BM at C = 43.33 x 2 - 50 x 2 = -73.32 kN, BM at A = 0

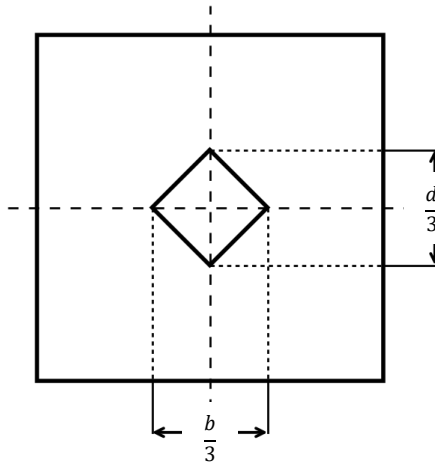
UNIT III – SIMPLE STRESSES AND STRAINS



2 Marks

Q.1. What is core or kernel of the section? Show the core or kernel of a rectangular section?

Ans – Core or kernel of the section is the section within which the load may be placed so as not to produce any tensile stress. Core or kernel of rectangular section is $B/3$ and $D/3$ in the direction of length (B) and depth (D) respectively.



Q.2. Define the term “bending stress”?

Ans – The bending moment at a section tends to bend or deflect the beam and the internal stresses resist its bending. The process of bending stops, when every cross section sets up full resistance to the bending moment. The resistance offered by the internal stresses to the bending is called bending stress.

Q.3. What is moment of resistance?

Ans – The compressive stresses and tensile stress from a couple, whose moment must be equal to the external moment (M). The moment of this couple which resists the external bending moment is known as moment of resistance.

Q.4. Where shall the maximum compressive stress develop, when a cantilever is loaded at its free end?

Ans – At bottom fibre, the maximum compressive stress shall develop, when a cantilever is loaded at its free end.

Q.5. What is the ratio of maximum shear stress to the average shear stress, when a rectangular section of a beam is subjected to a shearing force?

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS

Ans – The ratio of maximum shear stress to the average shear stress is 1.5, when a rectangular section of a beam is subjected to a shearing force.

Q.6. What is the value of shear stress at the top edge of the section, if a square with side x of a beam is subjected to a shearing force of ' F '?

Ans – The value of shear stress at the top edge of the section is zero, if a square with side x of a beam is subjected to a shearing force of ' F '.

Q.7. At what position the maximum shear stress will occur in an inverted T section subjected to a shear force F .

Ans – At neutral axis of the section, the maximum shear stress will occur in an inverted T section subjected to shear force F .

Q.8. Write the variation of bending stress in a beam having rectangular cross section?

Ans – Bending stress varies linearly having a maximum value at the extreme fibres and zero at the neutral axis.

Q.9. At what position the maximum shear stress will occur in a triangular section subjected to a shear force F .

Ans – Maximum shear stress will occur at the mid depth of the section i.e., $d/2$ from the apex.

Q.10. What is the section modulus of the circular section of diameter D .

Ans – The section modulus is $\frac{\pi D^3}{32}$

5 Marks

Q.1. State the assumptions made in the theory of simple bending?

Ans –

- a) The material of the beam is perfectly homogeneous (i.e., of the same kind throughout) and isotropic (i.e., of equal elastic properties in all directions).
- b) The beam material is stressed within its elastic limit and thus obey Hook's law.
- c) The transverse sections which were plane before bending remain plane after bending also.
- d) Each layer of the beam is free to expand or contract, independently of the layer above or below it.
- e) The value of E (Young's modulus of elasticity) is the same in tension and compression.
- f) The beam is in equilibrium i. e there is no resultant pull or push in the beam section.

Q.2. Prove the relations, $M/I = \sigma / Y = E / R$

Ans – $M/I = \sigma / Y = E / R$

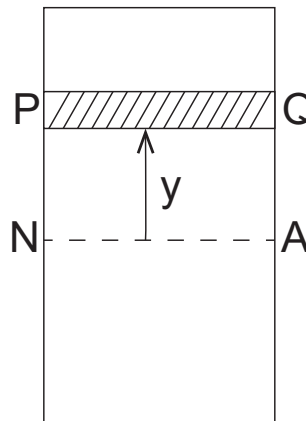
Where M = Bending moment.

I = Moment of Inertia.

σ = Bending stress in a fibre, at a distance Y from the neutral axis.

E = Young's modulus.

R = Radius of curvature.



UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS

Consider a section of the beam. Let NA be the neutral axis of the section. Now consider a small layer PQ of the beam section at a distance 'y' from the neutral axis.

Let σ_a = Area of the layer PQ.

The intensity of stress in the layer PQ.

$$\sigma = y \times \frac{E}{R}$$

$$\text{Total stress in the layer PQ} = y \times \frac{E}{R} \times \sigma_a$$

And moment of this total stress about the neutral axis

$$y \times \frac{E}{R} \times \sigma_a \times y = \frac{E}{R} \times y^2 \times \sigma_a$$

The algebraic sum of all such moments about the neutral axis must be equal to M.

$$\text{Therefore, } M = \sum \frac{E}{R} \times y^2 \times \sigma_a = \frac{E}{R} \sum y^2 \times \sigma_a$$

The expression $\sum y^2 \times \sigma_a$ represents the moment of inertia of the area of the whole section about the neutral axis.

$$\text{Therefore } M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R}$$

We have seen that

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\text{So } \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Q.3. A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 6m. If the beam is subjected to central point load of 12KN. Find the maximum bending stress induced in the beam section.

Ans –

Given Data -

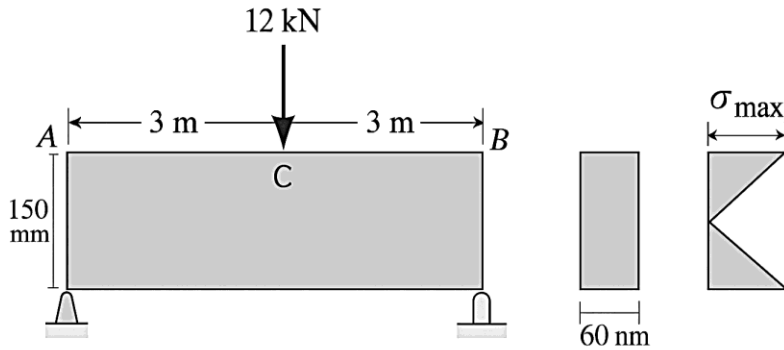
Width (b) = 60mm

Depth (d) = 150mm

Span (l) = 6 x 10³ mm

Load (W) = 12KN = 12 x 10³ N

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS



We know that the maximum bending moment at the center of a simply supported beam subjected to a central point load,

$$M = \frac{wL}{4} = \frac{(12 \times 10^3) \times (6 \times 10^3)}{4}$$
$$= 18 \times 10^6 \text{ N-mm}$$

And section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{60 \times 150^2}{6}$$
$$= 225 \times 10^3 \text{ mm}^3.$$

Maximum bending stress,

$$\sigma_{\max} = \frac{M}{Z}$$

$$\sigma_{\max} = \frac{180 \times 10^6}{225 \times 10^3} = 80 \text{ N/mm}^2$$

Q.4. A wooden beam 100mm wide, 250mm deep and 3m long is carrying a uniformly distributed load of 40KN/m. Determine the maximum shear stress and sketch the variation of shear stress along the depth of the beam.

Ans –

Given Data -

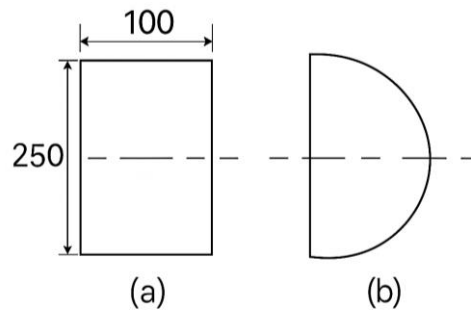
Width (b) = 100mm

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS

Depth (d) = 250mm

Span (l) = 3m = 3×10^3 mm

Uniformly distributed load (w) = 40kN /m = 40 N/mm



The shear force at one end of the beam, $F = \frac{wL}{2} = \frac{40 \times 3 \times 10^3}{2}$ N
 $= 60 \times 10^3$ N.

Area of the beam section, $A = b \times d = 100 \times 250 = 25000 \text{ mm}^2$.

Average shear stress across the section,

$$\begin{aligned}\tau_{avg} &= \frac{F}{A} = \frac{60 \times 10^3}{25000} \\ &= 2.4 \text{ N/mm}^2 \\ &= 2.4 \text{ MPa.}\end{aligned}$$

Maximum shear stress, $\tau_{max} = 1.5 \times \tau_{avg}$

Q.5. A circular beam of 150 mm diameter is subjected to a shear force of 7 kN. Calculate the value of maximum variation of shear stress along the depth of the beam.

Ans – Diameter of the beam, $D = 150\text{mm} = 0.15\text{m}$

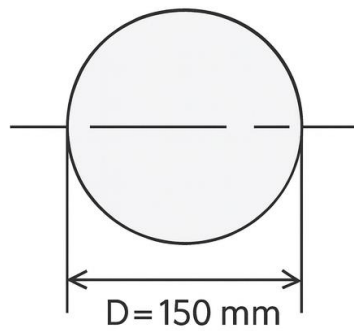
Area of cross section, $A = \frac{\pi D^2}{4} = 0.0176 \text{ m}^2$

Shear force, $F = 7 \text{ kN}$

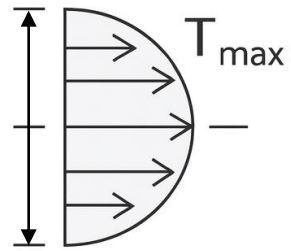
Average shear stress across the section, $\tau_{avg} = \frac{F}{A} = \frac{7}{0.0176} = 396 \text{ kN/m}^2$

Using the relation, $\tau_{max} = 4/3 \tau_{avg}$
 $= 4/3 \times 396$
 $= 528 \text{ kN/m}^2$.

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS



(i) Beam
cross-section



(ii) Shear stress
distribution

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS

10marks

Q.1. An I – Section, with rectangular ends, has the following dimensions.

Flanges = 150mm

Web = 300mm x 10mm

Find the maximum shearing stress developed in the beam for a shear force of 50 KN.

Ans –

Given Data:

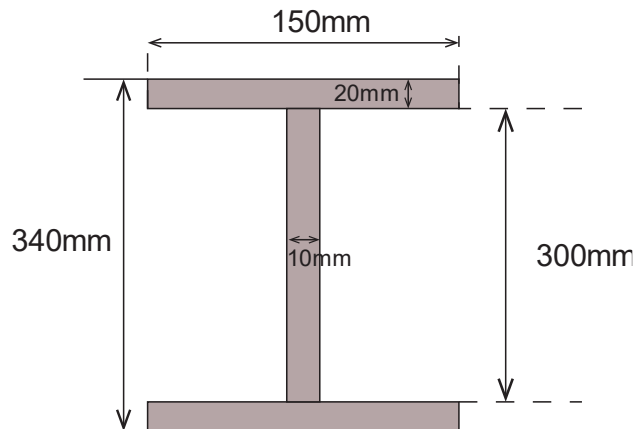
Flange width (B) = 150mm

Depth of web (d) = 300mm

Width of web = 10mm

Overall depth of the section (D) = 340mm

And shearing force (F) = 50 KN = 50×10^3 N



The moment of inertia of the I-Section about its center of gravity and parallel to x-x axis,

$$I_{XX} = \frac{150 \times 340^2}{12} - \frac{140 \times 300^2}{12}$$
$$= 176.3 \times 10^6 \text{ mm}^4.$$

Maximum shear stress,

$$\tau_{max} = \frac{F}{lb} \left[\frac{B}{8} (D^2 - d^2) + \frac{bd^2}{8} \right]$$

$$\frac{50 \times 10^3}{(176 \times 10^6) \times 10} \left[\frac{150}{8} (340^2 - 300^2) + \frac{10 \times 300^2}{8} \right] \text{ N/mm}^2$$

UNIT IV – BENDING AND SHEAR STRESSES IN BEAMS

$$= 16.8 \text{ N/mm}^2.$$

Q.2. A rectangular beam 60mm wide and 150 mm deep is simply supported over a span of 4m. If the beam is subjected to a uniformly distributed load of 4.5 kN/m, find the maximum bending stress induced in the beam.

Ans –

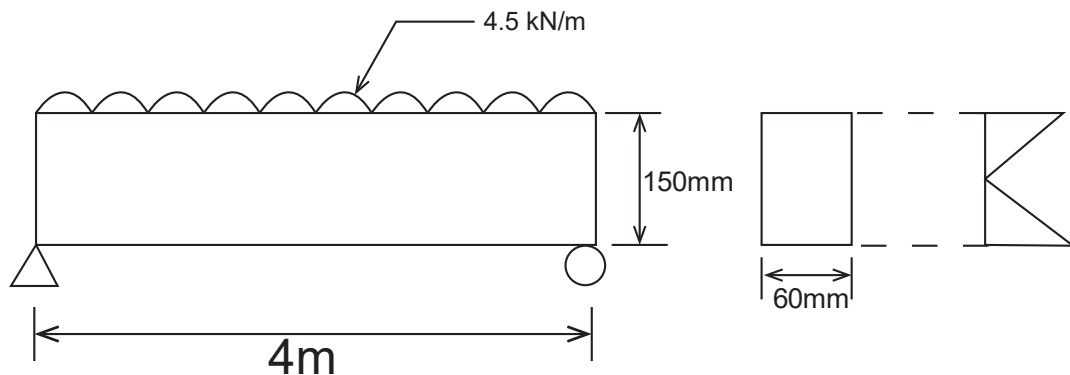
Given Data-

Width (b) = 60mm

Depth (d) = 150mm

Span (l) = 4m = 4×10^3 mm

Uniformly distributed load (w) = 4.5 kN/m = 4.5 N/mm



The section modulus of the rectangular section,

$$Z = \frac{bd^2}{6} = \frac{60 \times 150^2}{6} \\ = 225 \times 10^3 \text{ mm}^3.$$

The maximum bending moment at the center of a simply supported beam subjected to a uniformly distributed load

$$M = \frac{wl^2}{8} = \frac{4.5 \times (4 \times 10^3)^2}{8}$$

$$= 9 \times 10^6 \text{ N-mm}$$

$$\text{Maximum bending stress, } \sigma_{max} = \frac{M}{Z} = \frac{9 \times 10^6}{225 \times 10^3} = 40 \frac{\text{N}}{\text{mm}^2}$$

2 Marks

Q1. What is a compression member? Give one example.

Ans – A compression member is a structural member that is subjected to axial compressive force.

Example – Column in a building frame.

Q2. Define short column and long column.

Ans –

Short column is a column with slenderness ratio < 80 , fails by crushing.

Long column is a column with slenderness ratio > 80 , fails by buckling.

Q3. Define effective length of a column.

Ans –

Effective length is the equivalent length of an ideal column with pinned ends, which has the same buckling load as the actual column with given end conditions.

Q4. Define radius of gyration and give its formula.

Ans –

Radius of gyration (k) is the distance from the axis at which the entire area may be assumed to be concentrated for the same moment of inertia.

$$k = \sqrt{I / A}$$

Q5. Define slenderness ratio and mention its significance.

Ans –

Slenderness ratio (λ) is the ratio of effective length to least radius of gyration.

$$\lambda = L_e / k$$

It determines whether a column is short or long.

Q6. Write the four types of end conditions for a column.

Ans –

1. Both ends hinged
2. Both ends fixed
3. One end fixed and other free
4. One end fixed and other hinged

Q7. What is buckling of a column?

Ans –

Buckling is the sudden lateral deflection of a column under axial compressive load, occurring at a critical load called buckling load.

Q8. State the assumptions made in Euler's theory.

Ans –

1. Column is perfectly straight and homogeneous.
2. Load acts axially through the centroid.
3. Cross-section is uniform and small compared to length.
4. Stress is within elastic limit.

Q9. Write Euler's formula for buckling load of a column.

Ans –

$$P_{cr} = (\pi^2 \times E \times I) / (L_e)^2$$

where,

P_{cr} = Critical load

E = Young's modulus

I = Moment of inertia

L_e = Effective length

Q10. Define working load, design load, and factor of safety.

Ans –

Working Load: Actual load the member carries safely in service.

Design Load: Load used for design, given by (Working Load × FOS).

Factor of Safety (FOS): Ratio of ultimate load to working load.

5 Marks

Q1) A 150 mm diameter steel column, length $L = 3$ m, has both ends hinged (pinned). $E = 200$ GPa. Find the Euler critical (buckling) load and the safe load if $FOS = 3$.

Ans – For a circular section: $I = (\pi / 64) \times d^4$

$$d = 0.150 \text{ m} \rightarrow I = (\pi / 64) \times (0.150)^4 = 2.49 \times 10^{-5} \text{ m}^4$$

End condition: pinned–pinned $\Rightarrow L_e = L = 3 \text{ m}$

Euler's formula: $P_{cr} = (\pi^2 \times E \times I) / (L_e)^2$

$$P_{cr} = (\pi^2 \times 200 \times 10^9 \times 2.49 \times 10^{-5}) / (3)^2$$

$$P_{cr} = 5.45 \times 10^6 \text{ N} = 5450 \text{ kN}$$

Safe load ($FOS = 3$): $P_{safe} = P_{cr} / 3 = 1.82 \times 10^6 \text{ N} = 1817 \text{ kN}$

$$P_{cr} \approx 5450 \text{ kN}, \quad P_{safe} \approx 1817 \text{ kN}.$$

Q2) A 120 mm \times 200 mm solid rectangular column, length $L = 4$ m, is fixed at one end and free at the other. $E = 200$ GPa. Find the Euler buckling load (buckling about weaker axis).

Ans – Weaker axis \Rightarrow use smaller side $h = 120 \text{ mm} = 0.12 \text{ m}$

$$b = 200 \text{ mm} = 0.20 \text{ m}$$

Moment of inertia: $I_{min} = (b \times h^3) / 12$

$$I_{min} = (0.20 \times 0.12^3) / 12 = 2.88 \times 10^{-5} \text{ m}^4$$

End condition: fixed–free $\Rightarrow L_e = 2L = 8 \text{ m}$

Euler: $P_{cr} = (\pi^2 \times E \times I_{min}) / (L_e)^2$

$$P_{cr} = (\pi^2 \times 200 \times 10^9 \times 2.88 \times 10^{-5}) / (8)^2 = 8.88 \times 10^5 \text{ N} = 888 \text{ kN}$$

$$P_{cr} \approx 888 \text{ kN}.$$

Q3) A solid circular steel column of diameter $d = 100$ mm and length $L = 2$ m has both ends fixed. Find the slenderness ratio and classify the column.

Ans – For a solid circle: $k = \sqrt{I / A} = d / 4 = 100 / 4 = 25 \text{ mm}$

Both ends fixed $\Rightarrow L_e = L / 2 = 1 \text{ m} = 1000 \text{ mm}$

Slenderness ratio: $\lambda = L_e / k = 1000 / 25 = 40$

Since $\lambda < 80$, the column is short.

Q4) A 120 mm diameter solid circular column of length $L = 3.5$ m has both ends hinged.

Given: $\sigma_c = 320$ MPa, $a = 1 / 7500$, FOS = 2.5.

Find the Rankine crippling and safe loads.

Ans – Area: $A = (\pi / 4) \times d^2 = (\pi / 4) \times (0.12)^2 = 0.01131 \text{ m}^2$

Radius of gyration: $k = d / 4 = 0.12 / 4 = 0.03 \text{ m}$

$L_e = L = 3.5 \text{ m}$

$(L_e / k)^2 = (3.5 / 0.03)^2 = 13,611.11$

Rankine: $P = (\sigma_c \times A) / [1 + a \times (L_e / k)^2]$

$P = (320 \times 10^6 \times 0.01131) / [1 + (1 / 7500) \times 13,611.11]$

$P = 1.286 \times 10^6 \text{ N} = 1286 \text{ kN}$

Safe load: $P_{\text{safe}} = P / \text{FOS} = 1.286 \times 10^6 / 2.5 = 5.14 \times 10^5 \text{ N} = 514 \text{ kN}$

$P \approx 1286 \text{ kN}$; $P_{\text{safe}} \approx 514 \text{ kN}$.

Q5) State the effective length factors (K) for common end conditions and explain “effective length.”

Ans – Effective length L_e is the length of an equivalent pinned–pinned column having the same buckling load as the actual column.

It is given by: $L_e = K \times L$

End Condition	K (Value)
Both ends hinged	1.0
Both ends fixed	0.5
One end fixed, other hinged	0.7
One end fixed, other free	2.0

Hence, effective length $L_e = K \times L$ depends on end restraints.

10 Marks

Q.1) A hollow alloy tube 4m long with external and internal diameters of 40mm and 25 mm respectively was found to extend 4.8 mm under a tensile load of 60 kN. Find the buckling load for the tube with both ends pinned. Also find the safe load on the tube taking a factor of safety of 5.

Ans - Given:

Length, $L = 4\text{m}$

External diameter (D) = 40mm

Internal diameter (d) = 25mm

$\delta L = 4.8\text{mm}$

Tensile load = $P = 60\text{kN} = 60 \times 10^3 \text{ N}$

FOS = 5

Area of the cross section = $\frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4} \times (40^2 - 25^2) = 765.8\text{mm}^2$

Moment of inertia of the tube = $I = \frac{\pi}{4} (D^4 - d^4) = \frac{\pi}{4} (40^4 - 25^4) = 106500\text{mm}^4$

Using $\delta L = \frac{PL}{AE}$

$$\Rightarrow E = \frac{PL}{A \cdot \delta L} = \frac{60 \times 10^3 \times 4000}{765.8 \times 4.8} = 65291 \frac{\text{N}}{\text{mm}^2}$$

Since the column is pinned at both ends, therefore equivalent length $L_e = 4 \times 10^3 \text{ mm}$

Euler's buckling load = $P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi \times 65291 \times 106500}{(4 \times 10^3)^2} = 4290 \text{ N} = 4.29 \text{ kN}$

Safe load = $\frac{\text{Buckling Load}}{\text{FOS}} = \frac{4.29 \text{ kN}}{5} = 0.858 \text{ kN}$

Q.2) A hollow cylindrical steel column of 38 mm external diameter and 2.5 mm thick, having a length of 2.3m and hinged at both ends. Determine the crippling load by Rankine's formula using constants $\sigma_{cs} = 335 \text{ MPa}$ and $a = 1/7500$.

Ans –

Given:

External diameter (D) = 38mm

Internal diameter (d) = 38mm – 2 x thickness = 38mm - 2 x 2.5 mm = 33 mm

Length L = 2.3m

Since the column is pinned at both ends, therefore equivalent length $L_e = 2.3 \times 10^3 \text{ mm}$.

$$\sigma_{cs} = 335 \text{ MPa}$$

$$a = 1/7500$$

$$\text{Area of cross section} = \frac{\pi}{4} (D^2 - d^2) = \frac{\pi}{4} (38^2 - 33^2) = 88.75\pi \text{ mm}^2$$

$$\text{Moment of inertia, } I = \frac{\pi}{64} (D^4 - d^4) = \frac{\pi}{64} (38^4 - 33^4) = 14050\pi \text{ mm}^4$$

$$\text{Least radius of gyration, } k = \sqrt{\frac{I}{A}} = \sqrt{\frac{14050\pi}{88.75\pi}} = 12.6 \text{ mm}$$

$$\text{Therefore, Rankine's crippling load} = P_R = \frac{\sigma_{cs} \times A}{1 + a \left(\frac{L_e}{k} \right)^2} = \frac{335 \times 88.75\pi}{1 + \frac{1}{7500} \left(\frac{2.3 \times 10^3}{12.6} \right)^2} =$$

$$17161 \text{ N} = 17.16 \text{ kN}$$

***** END *****